CONTEXTUALIZED ABSOLUTIST-CONSTRUCTIVIST TEACHING APPROACH: EFFECTS ON STUDENTS' CONCEPTION, ATTITUDE, LEARNING EXPERIENCES, AND ACHIEVEMENT IN MATHEMATICS

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This paper prepared by **RAINILYN L. DUQUE** with the title **CONTEXTUALIZED ABSOLUTIST-CONSTRUCTIVIST TEACHING APPROACH: EFFECTS ON STUDENTS' CONCEPTION, ATTITUDE, LEARNING EXPERIENCES, AND ACHIEVEMENT IN MATHEMATICS** is hereby accepted by the Faculty of Education, U.P. Open University, in partial fulfillment of the requirements for the degree Doctor of Philosophy in Education (Mathematics Education).

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Biographical Sketch

A mathematics educator, Rainilyn Leonardo-Duque is a professor of mathematics for a decade and a half vis-à-vis pursuing further studies to expand her professional career. She obtained her baccalaureate degree of Bachelor in Secondary Education Major in Mathematics from Bulacan State University (BulSU), where she currently teaches tertiary students. She received various academic excellence awards at the University and graduated cum laude in 2006. She took her Master of Arts in Mathematics Education degree at the Philippine Normal University (PNU) and graduated in 2011. She is a faculty scholar supported by the Commission on Higher Education (CHED) and Bulacan State University, and she pursues her Doctor of Philosophy in Mathematics Education at the University of the Philippines Open University.

Having so much passion in research and writing, Mrs. Leonardo-Duque published several research articles in both the national and international fora. She also presented mathematics education research in various research conferences in the country and abroad. She actively engaged in research activities in both the undergraduate and graduate level in BulSU for years.

Mrs. Leonardo-Duque authored various mathematics modules for tertiary students, and co-authored mathematics textbooks for K-12 Senior High School students.

In her spare time, Mrs. Leonardo-Duque loves to paint and do arts crafts. She also enjoys travelling and taking on adventures.

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DEDICATION

To my love, my daughter Scarlet Therese,

my rock, my husband Alvin.

Also for Dad Ramon (+).

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ABSTRACT

The study aimed to contribute a contextualized perspective of mathematics and, consequently, its classroom implications for a better understanding of its nature, the genesis and developmental stages of mathematical concepts, and their applications.

The study made a comparative analysis of the effects of Strict Absolutist, Strict Constructivist and Contextualized Absolutist-Constructivist teaching approaches on students' mathematics conception, attitude, learning experiences and achievement. The study used the quasi-experimental three-group pretest-posttest research design. However, the mathematics achievement posttest was not administered due to the sudden class suspension and community lockdowns caused by the COVID-19 pandemic. The quizzes were used, instead, where each item in the mathematics achievement test was matched to an item in the quiz that measured similar concept or skill.

The results of the study revealed that the Contextualized Absolutist-Constructivist approach to mathematics teaching significantly improved students' mathematics conception, attitude, learning experiences and achievement.

Based on the results, it is recommended to provide students with learning opportunities that impart understanding of the nature of mathematics, its history and application, and to allow students to be creative and curious in mathematics.

Keywords: *Contextualized Absolutist-Constructivist, Strict Absolutist, Strict Constructivist, Nature of Mathematics*

Chapter I

INTRODUCTION

Background of the Study

Tracing the history of mathematics as a discipline, one can discover the unresolved debate (Ernest, 1991; Raju, 1999; Davidson & Mitchell, 2008; Wallis, 2006) of its nature and the appropriate method to deliver its content. Mathematics has not undergone major changes in recent years. The characterization of mathematics as objective, logical, and based on objects or truths outside of human involvement made it incorrigible and resistant to conceptual shifts that other disciplines underwent (Davidson & Mitchell, 2008).

The absolutist view of mathematics suggests that mathematical reality lies outside us, and all that can be done is to discover or observe it. Recent notions of strict absolutism go far beyond believing that mathematical objects cannot be accessed or understood through our senses, and it bears no necessary relation to the world in which we live. The absolutism orientation has proven quite resilient, and absolutists are not ready to give up the unique stability and universality of mathematical knowledge.

Nevertheless, any attempt in the past by educators and philosophers to reform and shape the absolutist or traditionalist paradigm put their ideas in rigid convention, and offered the other extreme view of mathematics as subjective, relative, and *fallible*[1]. The radical strict constructivism is an unconventional approach to the problem of knowledge and knowing. It starts from the assumption that knowledge – no matter how

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[1]**Fallibilism** is the epistemological thesis that no belief (theory, view, thesis, and so on) can ever be rationally supported or justified in a conclusive way. *Internet Encyclopedia of Philosophy*

it is defined – is in the heads of individuals, and that the thinking subject has no alternative but to construct what the person knows based on own experience (Von Glasersfeld, 1995). At its best, constructivism offers ways where we struggle to come to know and do mathematics, an important consideration in the philosophy of mathematics education. At its worst, as Stemhagen (2004) described, constructivism lost the ability to account for the seemingly unique characteristics of mathematics: its stability and universality.

The strict absolutist mathematics teachers or traditionalists – individuals who call for more rigor and a “back-to-basics” approach to mathematics – view mathematics as a given and ready-made truth to be passed on to students. This strict absolutist approach to mathematics teaching employs the prevalent traditional lecture-discussion method where students redo what the teacher demonstrates. On the other hand, the strict constructivists – individuals who advocate a more child-centered and applied approach to mathematics education – view individuals as the generators of knowledge. In this strict constructivist approach to mathematics teaching, minimal supervision is given to students who are encouraged to construct their learning. This strict polarization of views has made educators and practitioners choose which side to take, adapt, and put into practice. Lerman (1990) emphasized that “changes in mathematics education need to challenge fundamental assumptions about the nature of mathematics or else remain marginal in effect”. Thus, a proposition for a more harmonic interaction of these perspectives and the educational practice is needed.

This study aimed to offer an alternative perspective of mathematics that proposes the melding of the two opposing philosophies of mathematics, and consequently suggests classroom implications to better mathematics education. In this study, an extensive review of the history and philosophy of mathematics was done to justify the acceptance of the alternative Contextualized Absolutist-Constructivist perspective suggested in this study, as an adequate perspective of mathematics. Classroom implications of this alternative perspective were also highlighted to offer better mathematics teaching practices. To the knowledge of the researcher, the Contextualized Absolutist-Constructivist (AC) perspective to mathematics suggested in this study was neither written nor studied as reflected in the reviewed literature.

Contextualized perspective is putting theoretical constructs into contexts to link them to real-world practical applications. The Contextualized Absolutist-Constructivist perspective suggests that mathematics is a stable body of knowledge that can grow and expand in the face of newly discovered or created information. The study argues that previous mathematical truths are not discarded in the emergence of new knowledge, instead the domain specificity of an existing concept is expanded to cater to a new realm. Realm is a consolidated knowledge about a mathematical theory. (Carette, J., Farmer, W.M., Kohlhase, M., 2014). This contextualized perspective is created in the hope to offer a better view of mathematics that recognizes both its absolute stability and adaptability. Moreover, this contextualized perspective may offer a potential softening of the entrenched divide between the polarized views of mathematics. Classrooms implications of the contextualized perspective are realized when mathematical concepts taught are relevant to the learners' prior knowledge, interest and day-to-day environment. For example, a lesson on geometric transformations is taught to students. Students can be given a mathematical task where they can apply what they have learned in a real life setting.

This Contextualized Absolutist-Constructivist approach to teaching mathematics suggests that for mathematics education to prosper it might be useful to think of it as partly found and partly made. It highlights the significance of engaging students with learning opportunities that impart learning of the nature of mathematics, as well as encourage students to become creative in mathematics. Discussion about students' understanding of the nature of mathematics is scarce (Davison, D. & Mitchell, J., 2008; Rowlands, S., Graham T., & Berry, J., 2011). Understanding of the nature of mathematics includes the knowledge and understanding of the history, genesis, and role of mathematics. This also includes students' understanding of whether mathematics is an absolute static truth or a dynamic discipline capable of expanding and growing. In this study, mathematics conception pertains to students' understanding of the nature of mathematics and how they can effectively learn it. This is believed to engender learning the historical account of mathematical concepts which, in turn, can foster an understanding of the developmental nature of mathematics and the influence of human agency on its perpetuation.

Attitudes are assumed to be the precursors of mathematics learning behavior (Mazana, Montero, & Casmir, 2019; Arslan, Yavuz, & Karatas, 2014) and it determines students' integral behavior that can affect their learning of the subject. In this study, Azjen's ABC Attitude Model was used as basis in understanding students' attitude towards mathematics. Azjen (1993) conceptualized attitude as an amalgam of three separate measurable components: affect (A), behavior (B), and cognition (C). This is called the ABC Attitude Model or Tripartite Model. According to Azjen, affect is the emotional component consisting of feelings and emotions that are associated with the attitude subject, in this case, mathematics. The behavior is the action component consisting of ways and observable actions towards the attitude object while cognition is the mental component that consists of beliefs and perceptions one holds about the attitude object.

This study also delved on students' learning experiences and its possible effects to their mathematics achievement. Learning experiences in mathematics greatly affect how students view themselves as learners of mathematics and how they form their attitudes and emotions toward the discipline (Dalby, 2014). Hence, it is clearly important that students are provided with engaging learning experiences. The contextualized approach to mathematics teaching suggests that matching the student's prior knowledge with the teaching methodology and materials is an integral proactive step in bridging the gap in mathematics learning.

Statement of the Problem

The study aimed to determine the effects of Contextualized Absolutist-Constructivist mathematics teaching on students' mathematics conception, attitude, learning experiences, and achievement.

Specifically, this research sought answers to the following questions:

1. Is the mathematics conception, attitude, learning experiences, and achievement posttest mean scores of the students in the Contextualized Absolutist-Constructivist teaching approach group higher than that of the Strict Absolutist (conventional) and Strict Constructivist teaching approach groups?
2. Is the mathematics conception mean gain score of students in the Contextualized Absolutist-Constructivist teaching approach group higher than that of students in the Strict Absolutist and Strict Constructivist teaching approach groups in terms of

    a. understanding of the nature of mathematics; and

    b. beliefs about mathematics learning and its use?

3. Is the Mathematics attitude mean gain score of students in the Contextualized Absolutist-Constructivist teaching approach group higher than that of students in the Strict Absolutist and Strict Constructivist teaching approach groups in terms of

    a. affect dimension;
    b. behavior dimension; and
    c. cognitive dimension?

4. Is there a positive relationship between students'

    a. mathematics conception and attitude;
    b. understanding of the nature of mathematics and learning appreciation; and
    c. beliefs about learning mathematics and mathematics learning strategies?

5. Are students' mathematics conception, attitude and learning experiences positive predictors of mathematics achievement?

Significance of the Study

The teacher's beliefs and understanding of the nature of mathematics greatly affect what and how they teach the discipline. Thus, teachers who use the Contextualized Absolutist-Constructivist perspective of mathematics will have a deeper knowledge and understanding of the nature of mathematics. This is critical in delivering mathematical content to students, creating learning opportunities, and fostering an appreciation of the discipline among students. Through this study, mathematics teachers can reflect and understand their philosophy of mathematics, which is requisite for a paradigm shift.

The results of the study will benefit educators in having a better view of mathematics as a universal truth capable of evolving and growing from continuing human quest and exploration to improve living conditions. For example, the emergence of new geometric systems, such Spherical Geometry to improve ways and means of navigation. This new alternative perspective of mathematics can provide teachers with classroom ideas to influence students to view themselves as mathematics contributors capable of discovering, creating, and expanding mathematics concepts, as they better appreciate and understand the beauty and fullness of the discipline. Teachers will then understand that it is better to view mathematics as partly found and partly made. They can demonstrate an absolutist approach when teaching mathematics skills by allowing students to gain mastery with repeated mathematics drills; and a fallibilist perspective when engaging students in actual and realistic problem solving allowing them to explore, create and adjust their understanding.

School administrators, through this study may realize the significance of social context in creating a school climate that encourages collaboration, curiosity, and creativity in mathematics and realize the capacity of mathematics to expand and change. This will enable them to foster systemic change in the school rather than to expect one teacher to change philosophy or practice. Through the initiatives and the commitment of school administrators and authorities, inducing change in mathematics education which includes transforming mathematics methodology – from one that emphasizes the students' rote mechanical mathematics skill acquisition, to one that is enriched with discovery and invention – becomes possible. School administrators will concomitantly understand the importance of a teachers' training rich with a philosophical discourse that will provide them the opportunities to consistently reflect on their mathematics philosophy and practices to improve teaching-learning strategies.

Curriculum developers and educational authorities can become more aware of the importance of tailoring mathematics educational policies, programs, and curricula to the needs and capacities of the learners. Through their technical expertise in curriculum development and instructional leadership, the teaching and learning of mathematics become more collaborative, practical, and interactive.

Mathematics researchers can look more closely into students' understanding of what mathematics is and what it is for by exploring their conceptions of the discipline and finding connections of these conceptions with their performance in mathematics. Educational researchers can gain more insights into the relationship of personal mathematics philosophies of teachers and their effects on the learning of students.

Scope and Delimitation of the Study

The study does not present a new philosophy of mathematics but an alternative contextualized perspective of mathematics and consequently suggests contextualized pedagogical implications.

The sample consisted of three intact classes of freshmen BS Mathematics college students in Bulacan State University in Malolos City, Bulacan. The students were enrolled in the course, Fundamental Concepts of Mathematical Structure. Data were collected during the First Semester of the Academic Year 2019-2020.

The researcher taught the three classes covering the following topics which are based on the course syllabus: Propositions, Tautologies, and Logical Equivalence, Arguments and Proofs, Sets and Operations, Set Identities and Laws of Algebra of Sets, Divisibility and Euclidean Algorithm.

Chapter II

REVIEW OF RELATED LITERATURE AND CONCEPTUAL FRAMEWORK

This chapter presents the review of historical and philosophical views in mathematics, the analysis and justification of the Contextualized Absolutist-Constructivist perspective, and the suggested classroom implications of such perspective.

Related Literature

Absolutist-Constructivist: Contextualized Perspective

The Contextualized Absolutist-Constructivist perspective to mathematics in this study was used to seamlessly tailor the entrenched divide between mathematics philosophy to engender better mathematics teaching. In terms of teaching, the "melding of the absolutist and fallibilist perspectives, or philosophical movement within the arena of mathematics perspectives, may be essential for authentic learning to occur", (Davison & Mitchelle, 2008) thus ensuring that the combination of the two perspectives yields equitable form of utilizing philosophy and technical knowledge in teaching. The contextualized perspective to mathematics in this study, recognizes that mathematics is a stable body of knowledge that is certain, objective, and systematic, but can change, grow, and expand, engendered by continuous utilization and interaction of humans, to its realm.

The researcher used several theories of mathematics philosophy and mathematics education to establish a reasonable acceptance of the alternative approach. One of these is Kitcher's theory. Kitcher argued that the "originators of mathematics made their mathematical observations in the midst of trying to solve practical problems" (1983). His ideas of empiricism through the notion of perception as a critical component of the origins of mathematics puts into perspective the importance of mathematics history as an integral element to understand the concepts of mathematics as a subject. Kitcher further linked the contemporary practice to the origins of mathematics which brought both the history of mathematics and the influence of community into play. Understanding that the origins of mathematics can be accounted for as a useful and effective idea and tool to help men resolve problems gave Kitcher's theory relevance for the study, wherein the theory was used for the study's theoretical framework.

In a similar line of thought, Imre Lakatos proposed quasi-empiricism as a theory of mathematics that rejected absolute certainty as a goal and acknowledged the historical, hypothetico-deductive nature of mathematics (Ernest, 1991, p.34). He called his way of thinking of mathematics as "quasi-empiricism" to show that albeit science and mathematics are similar in method, both differ in content. And that unlike science – wherein the truths are verified not by physical objects or observations – mathematics' truths are verified by mathematical ideas. Lakatos further emphasized that "mathematical activity is human activity… mathematical activity produces mathematics", (Lakatos, 1976 as quoted in Ernest, 1991 p.37) which supported the concept of human influence in mathematics. This is further elucidated by Stemhagen in his paper: "The result is a version of mathematics that is respectful of its human history of improvement (or at least change) through an unending series of conjectures, counters, and reconfigured conjectures", (2004, p. 89). Thus, the verifiable concept that history is necessary to understand mathematics and its philosophy continues to stand.

As history plays an integral part in understanding the concepts of mathematics, Ernest (1991) enumerated and discussed the five theses that can be identified as quasi-empiricism: *(1)* Mathematical knowledge is fallible; *(2)* Mathematics is hypothetico-deductive; *(3)* History is central; *(4)* The primacy of informal mathematics is asserted; and *(5)* A theory of knowledge creation is included. These propositions are all important to the justification of the contextualized perspective to mathematics vis-à-vis classroom mathematics teaching implications suggested in this study.

As Ernest further extrapolated:

> Mathematicians are fallible and their products, including concepts and proofs, can never be considered final or perfect but may require renegotiation as standards of rigor change, or as new challenges or meanings emerge. As a human activity, mathematics cannot be viewed in isolation from its history and its applications in science and elsewhere (1991, p. 35)

Lakatos' philosophy closely resembled the falsificationist philosophy of science of Karl Popper which he well acknowledged. In the heart of Lakatos' philosophy of mathematics is a theory of the genesis of mathematical knowledge. The genesis of mathematical knowledge was a theory of mathematical practice that resolved the question of the history of mathematics. He meticulously explained the process which transformed private creations into accepted public mathematical knowledge. Quasi-empiricism can be seen to have great potential to offer a further account of the contextualized perspective.

Next, Dewey's pragmatic and psychological philosophy of mathematics is identified to make a stronger account of the empirical origins of mathematics. Dewey's views of the importance of bridging the gap between the individual's interest and experiences provides a firm ground for the arguments of the Contextualized Absolutist-Constructivist approach to teaching mathematics suggested in this study. This allowed his foundation to grow and to expand towards further inquiries within and outside the discipline provided a strong ground for the Contextualized Absolutist-Constructivist approach to mathematics teaching as suggested as a consequence of the contextualized perspective to mathematics built on in this study. The contextualized perspective to mathematics defines mathematics based on its use. The essence and correctness of mathematics are viewed by how well these concepts and theories help improve human life. This functional theory to mathematics philosophy is also evident in Dewey's stance of philosophy of mathematics, and consequently of mathematics education. According to Dewey, mathematics is a mental activity that involves objects we engage in to help us improve our lives. The contextualized perspective focuses on a mathematics philosophy and its consequent mathematics pedagogy implications emphasizing the functional and practical applications of the discipline. The contextualized perspective to mathematics argues the inclusion of the critical component of the role that human activity plays in mathematics development. Dewey had successfully melded psychology and philosophy with his pragmatic conception of how we know and what there is to know.

Dewey's conception of thought as a mental activity in the world is evidence of his more general pragmatic and experimental conceptions of mathematics and how we understand it. Dewey also worked on reconciling rationalism and empiricism to advance a philosophy of mathematics and mathematics education that emphasized how mathematics is a mental activity involving objects in the world to improve our lives. According to Dewey's pragmatic account, "our mathematics is what it is, to a certain extent, because of how we live our lives," (Stemhagen, 2004). To Dewey, mathematics is defined by its use.

The ideas of progressive absolutism were also considered to fit with the arguments of a perspective that regard the unique stability of mathematics and its developmental nature. The contextualized perspective to mathematics and consequently contextualized approach to mathematics education in this study argues that concepts remain absolute truths and universal until a human finds possibility and necessity for its domain validity to expand, so as to cater to the new knowledge discovered or created. The birth of new knowledge is made possible as a human learns mathematical truths, creates relationships, and uses it for applications to improve the way of living whenever applicable.

Similar to the ideas of progressive absolutism, contextualized perspective further argues that mathematical knowledge, proven to be true and absolute, is not discarded in the discovery and creation of new and wider mathematical theories. For instance, the concept of parallel lines in Euclidean geometry remains to be true and absolute, when the physical surface being dealt upon is a plane surface. However, since this concept cannot be true on a spherical surface, the domain of validity of the concept, which is the set that defines or yields mathematical truths, is expanded to cater to its new realm, thereby engendering knowledge creation. The human realization that the earth is spherical expanded plane geometric knowledge. This knowledge expansion gave birth to new geometric systems that have meaningful practical applications in navigation, physics, and other areas. The contextualized perspective to mathematics emphasizes that mathematical knowledge is not discarded in the face of new knowledge creation but refined, improved, and expanded.

The Contextualized Absolutist-Constructivist perspective in this study posited that mathematics can be regarded as partly found and partly made. It is the mind that processes, analyzes, and creates knowledge as individuals make meaning of their experience. However, these mental activities include manipulating and understanding concrete and absolute truths of mathematical concepts. According to Descartes, the mind cannot just create whatever knowledge it desires.

Non-absolutist perspective acknowledges how individuals and groups actively get involved in understanding mathematics instead of dealing with problems as how absolutists present them. Non-absolutist accounts of mathematics, such as versions and arguments of the constructivist perspective, tend to create new sets of problems. More often than not, they either strongly counteract propositions that are perceived as an overly rigid explanation of mathematics by offering subjectivist explanations that seem to ignore the significance of adequately recognizing the stability of mathematical knowledge.

Similar to Descartes, Kant was described as not completely subjectivist when it comes to the human construction of knowledge. According to Stemhagen (2004), Kant agreed that the mind's categories help to organize information from the senses in everyone's mind. However, he emphasized that the structures are themselves universal and absolute. This argument undergirds the propositions of this study that mathematics can be better viewed as partly found and partly made.

It is argued in this study that to foster better mathematics student learning, the nature of mathematics needs to be understood and embodied by mathematics teachers. Dewey argued that the demand for a teacher is two-fold. The teacher must have a thorough knowledge of the discipline and an awareness of those common experiences an individual has that can be utilized to lead them toward understanding (Stemhagen, 2004). Thus, the alternative approach to mathematics teaching suggested in this study recommends that absolute and universal mathematical concepts can be presented to students through interactive and engaging ways, allowing them to think through, analyze critically and relate meaningfully to their prior experiences and knowledge. Vygostky's (1986) Zone of Proximal Development (ZPD) can be a guiding theory in tailoring classroom activities to students' prior knowledge to bridge the gap between what is given and what students already know. The Contextualized Absolutist-Constructivist approach to mathematics teaching suggested in this study posits that for more meaningful and effective learning to occur, it is integral to build on students' prior knowledge and from there expand with more complex and complicated concepts. After acknowledging that certain mathematical truths should be mastered by students through repeated drills, the teacher should allow and encourage students to apply what they have learned to practical applications and their day-to-day experiences. This allows students to see relationships and connections between the formal and abstract mathematics concepts they learned and the world they live in which can engender deeper students' appreciation of mathematics and consequently encourage more responsible learning.

Mathematical knowledge can grow and expand when applied to different fields. Lakatos explained that a counterexample need not falsify a theorem but rather specify its domain of validity allowing for the knowledge to expand and grow. This sense of fallibilism means that any theorem can be "falsified" not in the sense of rejection but in the sense of revision. As a consequence of the contextualized perspective to mathematics suggested in this study, contextualized mathematics teaching approach argued that students can be encouraged to test and validate what they have learned within the context of their interests acting like young and fledgling mathematicians. For example, a student who learned about the parallel postulate of Euclid and wishes to test the concept on a spherical physical surface such as the globe may do so because he is very much interested in geography. The student may then realize that the concept is not valid when the physical surface is curved. Such knowledge will provide him the opportunity to reflect on the domain specificity of the concept he has learned. The student will gain a deeper understanding that the Parallel Postulate is not refuted but is revised based on the domain of validity. This method of providing students the opportunity to apply their acquired knowledge will also impart philosophical ideas of mathematics: That mathematics knowledge with its absolute and objective nature can grow and expand. Furthermore, the philosophy of mathematics may act as a vehicle for developing students' creative thinking (Jankvist & Iversen, 2013). It has been accepted that the philosophy of mathematics may assist students in their sense-making of mathematical concepts, ideas, and constructs such as theorems, definitions, and proofs.

Hence, the smooth melding of absolutist and fallibilist views suggested in this study is expected to foster deeper and more effective learning among students. The long-standing and "nightmarish" battleground of "math views" can now be reconciled or at the least toned down. This is succinctly asked in the Davison and Mitchell's 2008 paper, *How is Mathematics Education Philosophy Reflected in the Math Wars*: "could it not be that this teacher may demonstrate an absolutist philosophy when teaching math skills, but a fallibilist philosophy when engaging the students in bona fide problem-solving?", (p.56). Despite this more sharply divided than the Contextualized approach being suggested in this study, it nevertheless provides the same line of thought.

The contextualized perspective to mathematics suggested in this study aims to soften and meld the strict polarization of the philosophy of mathematics which then has important classroom implications to better the understanding of the discipline and the ways of learning it.

Important Role of Philosophy of Mathematics

The role of the philosophy of mathematics is to reflect on and provide an account of the nature of mathematics. As Ernest (1991) explains it:

> The philosophy of mathematics addresses such questions as: What is the basis for mathematical knowledge? What is the nature of mathematical truth? What characterizes the truths of mathematics? What is the justification for their assertion? Why are the truths of mathematics necessary truths?
>
> Ernest (1991, p. 3)

According to Ernest (1991), the philosophy of mathematics aims to provide a foundation for the certainty of mathematical knowledge. Hence, its main essence is to give a systematic and secure foundation for mathematical knowledge and its truth. This depends on a widely used assumption that the function of the philosophy of mathematics is to provide a certain foundation for mathematical knowledge, and is commonly referred to as foundationism[2].

[2]**Foundationism** is concerned with a secure foundation or basis of certainty. And its main rival is the coherence theory of justification which on the contrary suggests that no secure foundation is needed but the coherence strength of the pieces.

Paul Ernest provided a set of criteria for an adequate philosophy of mathematics found in his book, *The Philosophy of Mathematics Education.* According to him, a proposed philosophy of mathematics should account for the following: *(i)* Mathematical knowledge: its nature, justification, and genesis, (*ii)* The objects of mathematics: their nature and origins, *(iii)* The applications of mathematics: its effectiveness in science, technology, and other realms, and *(iv) Mathematical* practice: the activities of mathematicians, both in the present and the past (2004, p. 27). These criteria represent a contextualization of the role of mathematics which was used to assess the contextualized perspective of mathematics suggested in this study. This contextualized perspective has consequent significant classroom implications which were implemented and analyzed in this study.

Nature of Mathematics

It was argued that the role of the philosophy of mathematics is to adequately account for the nature of mathematics. This accounting of the nature of mathematics should include the "external" issues such as the history, genesis, and practice of mathematics, and the "internal" epistemological and ontological aspects, such as the justification of mathematical knowledge.

In general, knowledge is regarded as a justified belief. More accurately, Ernest discussed that prepositional knowledge consists of propositions that are accepted or believed, provided there are adequate grounds available for asserting them (Shepfler, 1965; Chisholm, 1966; Woozley, 1949). *A priori* knowledge consists of propositions that are asserted based on reason alone that consist of the use of deductive logic and the meaning of terms. This kind of knowledge is independent of observations of the world. In contrast, empirical or *a posteriori* knowledge consists of propositions asserted on the basis of experience and observation of the world (Woozley, 1949). Knowledge is then classified based on the grounds for its assertion.

Mathematics was regarded as the body of certain knowledge. Specifically, mathematical knowledge is classified as *a priori* knowledge – knowledge that consists of propositions that are asserted based on reason alone that is outside of the influence of the matters of the world (Ernest, 1991). The foundation of mathematical knowledge consists of deductive logic and definitions from where the grounds for asserting the truthfulness of mathematical propositions are based. The deductive method provides the warrant for the assertion of mathematical knowledge (Ernest, 1991). In general, mathematical knowledge consists of statements justified by proofs that stand on mathematical axioms and underlying logic.

These axioms are considered as basic truths that do not need justification beyond their self-evidence (Blanche, 1966). Mathematics is considered certain due to the process of asserting the following: First, the basic statements used in proofs are taken to be true. That is, mathematical axioms are assumed to be true for developing the system under consideration, mathematical definitions, or terms are considered true by *fiat*[3], and logical axioms are also taken as true. Secondly, the logical rules of inference allow nothing but truths to be deduced from them hence preserving truths. Based on these two grounds, every statement including conclusions in a deductive proof is true.

[3]**Fiat** is a sense of formal authorization or proposition

Thus, all mathematical theorems are certain truths because they are all drawn through deductive proofs. Based on this accounting, many philosophers claim that mathematics truths are certain and absolute truths. This account of mathematical knowledge has been the accepted fact for almost 2,500 years. However, this claim is no longer accepted. Euclid's axioms and postulates cannot be considered basic and incontrovertible truths. Negating or denying some of these axioms and postulates will result in contradiction. The denial of some of them, most notably what is called Euclid's Fifth Postulate or the Parallel Postulate leads to a nascent geometric knowledge system (non-Euclidean geometries). Beyond Euclid, modern mathematical knowledge also emerges which stands on sets of axioms that cannot be considered as basic universal truths such as the axioms of group theory or set theory (Maddy, 1984). In this perspective, all mathematical theorems are considered as certain truths.

Ernest (1994) realized that the issue of the relationship between philosophies of mathematics and mathematics education is the central problem of mathematics education. The nature of mathematics is central to the philosophy of mathematics education. This also has significant practical outcomes and implications for the teaching and consequently learning of mathematics. As Thompson explicitly states,

> "The observed consistency between the teacher's professed conceptions of mathematics and the way they typically presented the content strongly suggests that the teacher's views, beliefs and preferences about mathematics do influence their instructional practice (1984, p.125).

Hersh also wrote a pressing issue on the importance of proper accounting for the nature of mathematics:

> "The issue then, is not, what is the best way to teach, but what mathematics is all about… controversies about high school teaching cannot be resolved without confronting problems about the nature of mathematics" (1979, p.33).

However, it has been observed in the past that when the nature of mathematics is realized in practice, it will create conflicts that result in the siding to either of the two camps of absolutism or relativism. The following sections present a detailed accounting of these mathematics perspectives. The different facets of those perspectives were thoroughly discussed to elucidate the opposing views and beliefs that cause this entrenched divide. Moreover, the various educational implications of such clashing views were analyzed.

The Strict Absolutist Perspective: Incorrigible Truth

Absolutism, sometimes also called infallibilism or apriorism, is a family of related approaches and thinking constituting the predominant philosophy of mathematics (Ernest, 1991). In this view of mathematics, absolute, certain, and unchallengeable truth is rendered to the nature of mathematical knowledge. Confrey (1981) argues,

> "Concepts in mathematics do not develop, they are discovered… the previous truths left unchanged by the discovery of a new truth… mathematics proceeding by an accumulation of mathematical truths and as having an inflexible, a priori structure".

Mathematical knowledge has been held long before as universally absolute truth. As Ernest (1991) further emphasized: "Historically, mathematics has long been viewed as the paradigm of infallibly secure knowledge". This is undergirded by Kitcher (1983), "mathematics is the pinnacle of human knowledge, that it is used to judge other claims of knowledge, and that mathematical knowledge is not obtained by ordinary perceptual experience". Kitcher labels this general absolutist line of thought as *apriorism*[4].

Absolutists view mathematics as a unique branch of knowledge that offers certain truth that is permanent, incorrigible, and regarded as above all other knowledge. Around 2,500 years ago, Euclid and his colleagues constructed a magnificent and important logical structure called the *Elements* which was considered as the paradigm for establishing truths at least until the end of the nineteenth century. At the beginning of the twentieth century, the absolutist view of mathematical knowledge encountered controversies and attacks as several antinomies and contradictions in mathematics were discovered.

Absolutist mathematics has many different forms. Some conceive of mathematics as existing completely outside of the human influence and some versions consider only what is inside a tightly bounded mathematical system. This significantly shows evidence of the seemingly existing external-internal (Platonist) and found-made (formalist) dualism within absolutist philosophies of mathematics.

In other words, these objects and all the other properties that govern them will never change. According to Plato, the truths of mathematics are permanent and unchanging. Plato further argued that humans can only discover or unfold these objects and the relations among them. As Hardy explains,

[4] **Apriorism** pertains to knowledge or reasoning which proceeds from theoretical deduction and not from empirical observations.

> “According to Platonism… a mathematician cannot invent anything because it is all there already. All he can do is discover.”
>
> G.H. Hardy, from a Mathematician’s Apology

This view of Plato is tantamount to his idea that humans cannot create or invent mathematics. Philip Kitcher (1983) described that “Platonists regard mathematical statements as descriptive of a realm of mind-independent of abstract objects” (p. 6). This was further elaborated by V. H. Klenk (1976), “the object exists independently of the human mind or mathematical system, and it is the job of the mathematician to discover or uncover their properties and relations” (p. 8). These lines of thought ultimately suggest that mathematics is void of human influence and that all he can do is to utilize it.

Plato has introduced the dualistic way of thinking, which may have influenced how objects or ideas are viewed. Plato has a notion of existence that is a two-level vision of reality as illustrated by pairs of unequal opposites (Press, 1999). Plato’s dualistic outlook juxtaposes body-soul, appearance-truth, belief knowledge, the changing-the unchanging, the senses-the intellect, and others. Furthermore, Plato argues that mathematical objects belong to the eternal realm and not in the physical realm that is ephemeral in the state.

Modern-day mathematical Platonists agree with Plato’s belief in the distinction between what our senses can perceive as mathematical objects and the “real” mathematical objects that exist in an abstract realm. However, according to modern notions of mathematical Platonism, as presented and dismissed by Hersh, Kitcher, and others, mathematical objects cannot be accessed and understood through our senses.

This recent notion of modern-day Platonists seems to take Plato in a literal way. Karen Parshall in a correspondence explained this dilemma: "(According to Plato) We have no chance of comprehending the Forms without our senses as an intermediary, BUT we have to be careful to use our intellect to help 'filter' the 'impurities' that our sense may suggest" (2002).

Platonism, albeit a widespread and most believed thinking of mathematics, cannot resolve many of its paradoxes. As Kline (1980) thesis,

> "Mathematicians and philosophers of mathematics was not willing to give up the certainty of mathematics and, in the face of what was perceived as the unsettling implications of the development of some new types of mathematics, they turned to formalism in hopes of preserving the indubitability of mathematics."

Hence, Platonists' external body of truths through the emphasis on the internal consistency of the truths leads to another form of absolutism in mathematics, formalism. The Platonists concern themselves with external content while the formalist primarily worries about internal rules of mathematics which clearly show that the dualistic approach to mathematics exists. The Platonism-formalism distinction proves that several forms of untenable dualism hinder our views to perceive mathematics in productive and more meaningful ways because of being confined with either of the two views.

Formalism is the perception of mathematics that explained the futility of formal games played with marks on paper, following rules. The formalist thesis comprises two claims as Ernest (1991) explains: (1) Pure mathematics can be expressed as uninterpreted formal systems, in which the truths of mathematics are represented by formal theorems; and (2) The safety of these formal systems can be demonstrated in terms of their freedom from inconsistency, through meta-mathematics. According to them, mathematics bears no necessary relation to the world in which we live.

The main difference between Platonism and formalism is that the former posits that mathematical objects exist in an ideal realm that is immutable and unchanging; while the latter regards mathematics as operations with symbols in each system. Furthermore, the focus of formalism is internal consistency as a means of legitimacy. According to Morris Kline (1980) as explained in his *Mathematics: The Loss of Certainty,* formalism questioned the certainty of the Platonic understanding of the nature of mathematics. Platonists enthroned mathematical certainty as to its idea, but formalists generally put importance on the certainty of results after some mathematical manipulations of symbols following accepted and formalized rules which is similar to playing games.

The Platonist's external focus and belief in pre-made mathematical objects and the formalist's internal focus and tendency to look for truth as the result of following rules has a clear distinction between content and method. These product-process dimensions of the two types of absolutists will also help in understanding each perspective as each presents an understanding of mathematics consisting of elements that are essentially outside of the human influence.

Hersh (1997) explains eloquently what seems to be the unsettling issue of these philosophies in what he calls the "mathematician's philosophical dilemma":

> The working mathematician is a Platonist on weekdays, a formalist on weekends. On a weekday, when doing mathematics, he is a Platonist, convinced he is dealing with an objective reality whose properties he is trying to determine. On weekends, if the challenge is to give a

> philosophical account of this reality, it is easiest to pretend he does not believe in it. He plays, formalist, and pretends mathematics is a meaningless game.

The absolutist perception and orientation have proven itself to be quite resilient and enduring because they can be useful in some regards. Morris Kline (1980) also emphasized that mathematicians and philosophers of mathematics were not willing to give up the certainty of mathematics. When they are in the face of what was perceived as the unsettling implications of the development of some new types of mathematics, such as the emerging new theories of geometry mentioned earlier, they turned to formalism in the hope of preserving the indubitability of mathematics.

The recent version of formalism is logicism which is frequently considered a freestanding philosophy of mathematics (Ernest, 1990; Stroll, 1999). It is the school of thought that regarded pure mathematics as a part of logic (Ernest, 1991), which has two claims: *(1)* All the concepts of mathematics can ultimately be reduced to logical concepts, provided that these are taken to include the concepts of set theory, or some system of similar power; *(2)* All mathematical truths can be proved from the axioms and rules of inference of logic alone.

According to Stroll, "all mathematical propositions (can) be reduced to propositions containing only logical concepts such as constants, quantifiers, variables, and predicates". For formalists, mathematics can be regarded as operations on a given set of symbols.

It has been accepted that Whitehead and Russell (1910-13) were able to use chains of definitions to establish the first of the two claims, however failed to do the same on the second. As Russell, admittedly explains it himself:

> But although all logical (or mathematical) propositions can be expressed wholly in terms of logical constants together with variables, it is not the case that, conversely, all propositions that can be expressed in this way are logical. We have found so far a necessary but not a sufficient criterion of mathematical propositions. We have sufficiently defined the character of the primitive *ideas* in terms of which all the ideas of mathematics can be *defined*, but not the primitive *propositions* from which all the propositions of mathematics can be *deduced.* This is a more difficult matter, as to which it is not yet known what the full answer is.
>
> We may take the axiom of infinity as an example of a proposition which, though it can be enunciated in logical terms, cannot be asserted by logic to be true.
>
> (Russell, 1919 as stated in Ernest, 1991)

Russel explained that not all mathematical theorems or propositions and consequently not all the truths of mathematics are derived from the axioms of logic alone. Thus, the second claim is refuted which then clearly shows that logicism fails to be a mathematics philosophy.

Ernest (1991) further explained that “logic was considered to provide a certain foundation for truth, apart from over-ambitious attempts to extend logic. Thus, if carried through, the logicist program would provide certain logical foundations for mathematical knowledge, reestablishing certainty in mathematics” (p. 9).

The modern and more liberated absolutist philosophy of mathematics emerged to recognize the ability of mathematical knowledge to change and grow. This may seem the beginning of a more open absolutism view of mathematics proposing the revision or expansion of the mathematical domain where the knowledge holds.

Recent forms of absolutism emerged which cater to the idea that knowledge in mathematics can change. As emphasized in Problems *with Fallibilism as a Philosophy of Mathematics Education*,

> "Providing the theorem follows from the axiom without error, the revision sets out the domain in which the theorem can be said to hold."
>
> - Rowlands (2011)

This argument is based on the reality of knowledge growth. Confrey suggests a more uncompounded way of thinking of mathematics and moves away from the formal absolutist philosophy of mathematics. As Confrey explains,

> "Progress is a process of replacement of previous theories by superior theories which account for all the previous data and more. Each progressive theory approximates truth more and more precisely… progress consists of discovering mathematical truths which are not consistent with a theory or not accounted for in the theory, and then extending the theory to account for this larger realm of mathematical phenomenon."
>
> Confrey (1981)

The key distinction is between static and dynamic absolutist conceptions of mathematical knowledge and theories, with human activity contributing to the dynamic in progressive absolutism. Formalism and logicism are formal absolutisms that view mathematics as consisting of no more than fixed, formal, and mathematical theories. In contrast, progressive absolutist philosophies: *(i)* accommodate the creation and change of axiomatic theories; (*ii)* acknowledge that more than purely formal mathematics exists, for mathematical intuition is needed as the basis for theory creation; and hence *(iii) acknowledge* human activity and its outcomes, in the creation of new knowledge and theories.

Intuitionism (and constructivism, more generally) is nicely related to a new perspective of a contextualized approach to mathematics. Intuitionism (*1)* acknowledges human mathematical activity as fundamental in the construction of proofs or mathematical objects, the creation of new knowledge, and *(2)* acknowledges that the axioms of intuitionistic mathematical theory (and logic) are fundamentally incomplete, and need to be added as more mathematical truth is revealed informally or by intuition (Brouwer, 1927; Dummett, 1977).

Intuitionism and progressive absolutist philosophies acknowledge human agency, albeit in a stylized form which is generally in the domain of informal mathematics satisfy more the adequacy criteria for a philosophy of mathematics compared to formal absolutist forms. These views were utilized in establishing a contextualized perspective to mathematics that recognizes the universality of mathematical truths and their ability to expand and grow.

At the other end of the stick is the philosophy of mathematics called constructivism. The non-absolutist theories that were discussed in the next sections generally see the inadequacy of absolutist notions of the nature of mathematics. However, the arguments of non-absolutist philosophers also tend to conceive mathematics as an internal-external division.

Non-Absolutist Mathematics: The Strict Constructivist Approach

The most general argument of non-absolutist philosophers is their strong opposition to the absolutist perspectives. Hersh (1979) explains that “it is reasonable to propose a new task for mathematical philosophy: not to seek indubitable truth but to give an account of mathematical knowledge as it is – fallible, corrigible, tentative and evolving, as is every other kind of human knowledge" (p. 43).

Ernest (1991), a philosopher of mathematics education, states: "The absolutist view of mathematical knowledge has been subjected to a severe, and in my view, irrefutable criticism. Its rejection leads to the acceptance of the opposing fallibilist view of mathematical knowledge. This is the view that mathematical truth is fallible and corrigible and can never be regarded as beyond revision and correction" (p. 18).

It is constructivism that became the most influential form of non-absolutism. Catherine Fosnot defines her general notion of constructivism in her book *Constructivism: Theory, Perspectives, and Practice.* It is worthwhile to quote a part of its preface for it discusses some of the crosscurrents and controversies within the philosophy:

> "Constructivism is a theory about knowledge and learning; it describes both what "knowing" is and how one "comes to know". Based on work in psychology, philosophy, and anthropology, the theory describes knowledge as temporary, developmental, nonobjective, internally constructed, and socially and culturally mediated. Learning from this perspective is viewed as a self-regulatory process of struggling with the conflict between existing personal models of the world and discrepant new insights, constructing new representations and models of reality as a human meaning-making venture with culturally developed tools and symbols, and further negotiating such meaning through cooperative social activity, discourse, and debate.
>
> Fosnot (1996)

Piaget also argued that the stage theory elucidates well his recognition that when an individual cannot fit new knowledge into existing schemes, he will seek equilibrium by constructing new levels of cognitive structures. This process is made possible by moving from one stage to another until he has reached reconciliation with the new knowledge from his sensory experiences and his existing knowledge structure. Furthermore, Piaget contended that an individual constructs his understandings through actively engaging with his physical environment. Piaget albeit a constructivist, also regarded that individuals acquire mathematical concepts through “unfolding logic” present in them. Piaget accepted an absolutist view of knowledge, especially mathematics (Ernest, 1991). Criticism of the views of Piaget extends to his focus on how each individual personally constructs an understanding of their world.

Ernst Von Glasersfeld posited what he considered a continuation of the Piagetian theory of cognitive development. He gave more emphasis on the solitary nature of knowledge construction and move away from Piaget’s claim of universal logical structures. Glasersfeld (1991) offered the idea that knowledge is the result of a learner’s activity rather than of the passive reception of information or instruction. He further elucidates how cognition helps: “us to cope in the world of our experience, rather than the traditional goal of furnishing an ‘objective’ representation of the world as it might ‘exist’ apart from us and our experience”.

However, what seemed to be the subjective view of knowledge of Von Glasersfeld tends to give some interesting light to the realm of mathematics as was debated by many before him. He gave an example, “the idea of an equilateral triangle”: A teacher discusses an equilateral triangle and draws one on the chalkboard. After further descriptions of what it means for a triangle to be equilateral, the students draw on their desks their equilateral triangle. Albeit the teacher and the students all may have understood what an equilateral triangle is and drawn an example of it, Glasersfeld (1995) emphasized that there exists no objective truth and none of the figures are examples of an equilateral triangle. “The real equilateral triangle, such a structure exists nowhere, except in your heads”. However, the demonstration and transfer of knowledge in the example given by Glasersfeld seems to contradict his theory that individuals construct their knowledge. Despite his comments that the structure exists nowhere but only in the heads of the thinkers, it moves him apart from the Platonist understanding of the mathematics realm since Platonists posit that objects exist in a God-given realm.

On the other hand, a threatening premise for the epistemology of mathematics comes in the form of radical constructivism. It stresses that the individual and the idiosyncratic processes of how individuals construct their knowledge creates the impossibility of meaningful connections between individuals.

Paul Ernest’s works were also considered in this study. His *Philosophy of Mathematics Education* is a comprehensive account of his works and research agenda. As Kurt Stemhagen (2004) discusses in his research paper *Beyond Absolutism and Constructivism: The Case for an Evolutionary Philosophy in Mathematics, a* social constructivist theory is primarily supported by three pillars: the radical psychological constructivist’s preoccupation with individual subjective knowledge, the focus on the import of the social and linguistic realms to mathematics, and the quasi-empirical depiction of mathematical knowledge as emerging through acts of human-led creation which were discussed on the latter part of this section.

Ernest (1991) explained that social constructivism views mathematics as a social construction. It draws on conventionalism, in accepting that human language, rules and agreement play a key role in establishing and justifying the truths of mathematics. Furthermore, he emphasized that social constructivism is descriptive as opposed to a prescriptive philosophy of mathematics, aiming to account for the nature of mathematics understood broadly, as in the adequacy criteria of a philosophy of mathematics. He further stated the grounds for describing mathematical knowledge as social construction: *(1)* The basis of mathematical knowledge is linguistic knowledge, conventions and rules, and language is a social construction; *(2)* Interpersonal social processes are required to turn an individual's subjective mathematical knowledge, after publication, into accepted objective mathematical knowledge; *(3)* Objectivity itself will be understood to be social.

Similar to von Glasersfeld's radical constructivism, Ernest expressed that individuals create or construct their private knowledge or understandings of mathematics to which a set of constraints and corrections to this individual knowledge construction is offered by the public and social realm.

Criticisms which come from traditions thrown at Ernest enabled him to draw his mathematical social constructivism theory. He opted to draw a sharp distinction between the private-public or inner-outer realms, instead of offering a combination of psychological and social constructivism. Ernest talked about the possibility of objective Truth (capital letter included) which shows that he is adopting the language of his opponents (absolutists). He also focused on the adoption of rules and ways of thinking within the community of mathematics which makes it evident that he is leaving behind the question of how such rules came to be.

Lev Vygotsky is regarded highly for his role in social constructivism. His fresh perspective in his work helped to greatly facilitate innovations in mathematics and mathematics education.

Vygotsky suggested the idea of learning within the zone of proximal development (ZPD). He argued that individual learns best when the concept presented is slightly above the individual's knowledge and capacity. He emphasized that the individual can be prompted to think critically when he can relate the concept to what he already know. This idea of Vygotsky is similar to the line of thought of Ausubel (1982) when he suggested the concept of Meaningful Learning Theory. According to this theory, as new information is incorporated into prior knowledge and concepts meaningful learning occur as new information is understood. Ausubel (1977) argued in similar thought that effective learning occurs when the concept is related to the previously learned knowledge of the individual.

Furthermore, Vygotsky differentiated between what he called pseudoconcepts and scientific concepts. He explained that pseudoconcepts are the naturally constructed or spontaneous concepts that individuals form as they reflect on their experiences (Fosnot, 1996). These structured ideas came from the teachings in his classrooms.

Despite the many implications of Vygotsky's work in terms of mathematics and mathematics education which are currently active, some concerns emerged on Vygotsky's theories. Many constructivists concern themselves about the degree to which culture determines everything from behavior to what individuals can think.

As mentioned above, one of the pillars of Ernest's social constructivism is the quasi-empirical depiction of mathematical knowledge as emerging through acts of human-led creation. One of the major influences in Ernest's works of constructivism is quasi-empiricism as a philosophy of mathematics of Imre Lakatos. Quasi-empiricism in mathematics then models itself on how we use the physical world to conjecture about scientific knowledge (Lakatos, 1976; Popper, 1959 as quoted in Ernest, 1991).

Having reviewed the different views and arguments, it is more than enough to say that both absolutism and constructivism, albeit have much innovation and meaningful ideas to offer, ultimately can fail as philosophies of mathematics. Absolutism suggests an understanding of mathematics that accounts for its unique stability but fails to acknowledge the influence of human activity and interaction in the accounting and creation of mathematics. Conversely, strong constructivism tends to emphasize an understanding of mathematics that highlights human involvement but, in doing so, seems to disregard the remarkable stability and universality of mathematical knowledge.

Stemhagen (2004) suggested an evolutionary approach that tends to acknowledge the functional nature of mathematics instead of focusing on its structural elements. In his paper, he argued that mathematical knowledge can be recreated. He explained, "the work of mathematicians from any era are, in a sense, time capsules, presenting much valuable information about the communities within which they lived and worked. This perspective is also evident in Philip Kitcher's (2003) understanding of mathematical knowledge.

However, Kitcher seems to have a rigid understanding of rationality. He explained that mathematical developments are: "rational insofar as they maximize the chances of attaining the ends of inquiry" (1988, p. 304). He sounded as though he viewed the "attainment of understanding" as the destination.

Tantamount to saying that a time when inquiry is ended will come because understanding has finally been reached. To truly advance towards an evolutionary theory of the nature of mathematics, it is best enough to recognize the influence of contingent events from both within the community of mathematicians and from outside of it.

Stemhagen's evolutionary approach accounted for the functional aspect of mathematics, which suggested that the "correctness" of a mathematical idea is according to how well it helps the holder of the belief achieve her desired end. This although may apply for basic arithmetic concepts of mathematics but not for more complex and abstract knowledge. The evolutionary perspective suggested by Stemhagen seems to be concerned only too much about the practical use of mathematics as he emphasized that the individual must test his knowledge in terms of the job that the mathematical knowledge was created to do. This seemed to be altogether devoid of any recognition of the unique stability of mathematical knowledge as each individual adjusts his knowledge of mathematics to how it serves him. On the other hand, the contextualized perspective to mathematics suggested in this study recognizes the unique stability and universality of mathematical truths and concomitantly its ability to expand and grow through domain specificity revision.

The following table presents a comprehensive comparison of the three perspectives: Strict Absolutist, Strict Constructivist and Contextualized Absolutist-Constructivist perspective in mathematics and their consequent classroom implications.

Table 2.1
Comparison of Strict Absolutist, Strict Constructivist and Contextualized Absolutist-Constructivist Perspectives and their Classroom Implications

Aspect	Absolutist	Constructivist	Contextualized
Nature of Mathematics	Mathematical knowledge is absolute truth, objective, logical,	Mathematical knowledge is subjective, tentative, relative and fallible.	Mathematical knowledge is stable, objective (absolutist) and capable of growing and evolving

	permanent and incorrigible.		(constructivist) - the specificity of the domain validity of the concept is expanded to cater to new mathematical knowledge (added layer of contextualized).
Learning of Mathematics Knowledge	Mathematical reality lies outside us and all that man can do is to discover it.	Mathematical reality is in the head of the thinking subject and that all he can do is to construct what he knows on the basis of his experience.	Mathematical reality is a mental activity (constructivist feature) where concrete and absolute truths (absolutist) is accessed and understood through the senses to make meaningful relations (contextualized).
Mathematics' relation to the world we live in	Mathematics has no necessary relation to the world we live in.	Mathematics helps us cope in the world we live in.	Mathematics is a functional tool to better understand the world (contextualized).
Mathematics' human agency	Mathematics exists outside of human influence.	Mathematics is a human-meaning making venture.	Mathematics is produced by human activity.
Learning of Mathematics Knowledge	Mathematics is a given and ready-made truths to be passed on to students.	Students construct their own mathematics understanding. They are the generators of knowledge.	Acknowledges the objective formal mathematics (absolutist) which are used for mathematical intuition and thinking processes (constructivist).

Classroom Implications			
Students' Learning Needs	Mathematics classroom practice is one-size-fits-all.	Students adjust their knowledge structures to the given mathematical problem.	Teachers values appropriate matching of content (contextualized) and the individual's capacity for a meaningful interaction and construction of understandings (constructivist).
Mathematical solution	Mathematical solutions are constructed based on the teacher's demonstration.	Exploration and discovery of mathematical solutions are encouraged.	Individualized mathematical solutions and methods are encouraged (contextualized).
Teaching the Nature of Mathematics	Nature of mathematics understanding and history of mathematics are hardly imparted.	Implied exposing of students to the nature of mathematics.	Explicit teaching of the nature of mathematics and rich incorporation of mathematics historical account are highlighted (contextualized).

Students' Conception of the Nature of Mathematics

The nature of mathematics has gained much attention over the last few decades resulting in more discussions about it (Dossey, 1992; Fuson, Kalchman, & Bransford, 2005; Winter, 2001; Presmeg, 2002; Ocean, 2005).

It was discussed in the previous sections of this chapter that the teachers' understanding of the nature of mathematics affects in one way or more their mathematics teaching practices and decisions. They can communicate subtle messages to students about their understanding of the nature of mathematics which in turn affects the way students view mathematics and its role in the world in which they live (Dossey, 1992). It can further enable or constrain students' process of linking everyday practices and what they learn from their mathematics class (Presmeg, 2002).

One of the different dichotomies of mathematics views, as focused in this study, is to distinguish between external conceptions of mathematics that view mathematics as a fixed body of knowledge that is presented to students, and the internal conceptions held by those who believe in mathematics as a personally constructed internal knowledge.

Recent research focused on analyzing the importance of knowing students' varying and unique perspectives of mathematics in leading them to better learning and appreciating the discipline (Fielding, Fuller, & Loose, 1999; Rudduck & Flutter, 2000; Young-Loveridge, 2005; Young-Loveridge & Taylor, 2005; Young-Loveridge, Taylor, & Hawera, 2005). A few studies delved into knowing how students view the mathematics that they do; however, most of these were done with very young students (Grootenboer, 2003; Howard & Perry, 2005; Masingila, 2000). The results of these studies revealed that most students position themselves as a passive recipient of knowledge. Moreover, these students seem to lack awareness of their mathematics competencies, capabilities, and problem-solving strategies – adding that it is most important to watch and listen to the teacher for them to learn (Howard & Perry, 2005). Also, students revealed a very narrow conception of the nature and purpose of mathematics which are limited mostly to number concepts and arithmetic (Grootenboer, 2003). Most students had difficulty talking about the nature of mathematics which suggests that many students do mathematics without much thought or opportunity to discuss what it is (Young-Loveridge, Sharma, Taylor, & Hawera, 2005).

Recent researches on epistemological beliefs or an individual's idea of the nature of knowledge and learning in general show that those beliefs are highly related to learning and understanding. In mathematics context, it was found out that the more students believe knowledge is certain the more likely they will find absolute answers and try to distort or ignore tentative information (Kardash & Scholes, 1996). And when students believe that knowledge is organized as isolated bits, the more difficulty they will have in understanding mathematical texts (Schommer et al., 1992). Moreover, when students believe that learning is quick they are more likely to have difficulty in comprehending and summarizing academic texts (Schommer, 1990).

These results of the review reveal the many problematic issues to the lack of understanding of the nature of mathematics, its purpose, and how it is best learned. If students lack sufficient understanding and consequently appreciation of what mathematics is all about then the linking of the mathematics they learn and what they encounter outside their classroom will be difficult if not impossible.

Students' Attitude Towards Mathematics

Attitude is the driving force that determines one's actions, beliefs, emotions, and motivation. It also pertains to the liking or disliking of the subject; tendency to engage in or avoid mathematical tasks; the general belief of one's mathematical ability (Kibrislioglu, 2016). It is evident from the literature that attitude determines students' integral behavior and ways that can affect their learning of the subject. Attitudes are assumed to be the precursors of mathematics learning behavior (Mazana, Montero, & Casmir, 2019; Arslan, Yavuz, & Karatas, 2014). Several studies revealed that students have been developing a negative attitude towards mathematics since the early years of their lives and this decreases more as they move towards high-grade levels (Kibrislioglu, 2015; Mazana, Montero, & Casmir, 2019; Arslan, Yavuz, & Karatas, 2014). Therefore, teachers should devise strategies and design various activities which could engender meaningful positive learning experiences for students to improve their attitude towards the discipline.

Azjen (1993) conceptualized attitude as an amalgam of three separate measurable components: affect (A), behavior (B), and cognition (C). This is called the ABC Attitude Model or Tripartite Model. According to Azjen, affect is the emotional component consisting of feelings and emotions that are associated with the attitude subject, in this case, mathematics. The behavior is the action component consisting of ways and observable actions towards the attitude object while cognition is the mental component that consists of beliefs and perceptions one holds about the attitude object.

Students' Mathematics Learning Experiences

The students' learning experiences in mathematics greatly affect how they view themselves as learners of mathematics and how they form their attitudes and emotions toward the discipline (Dalby, 2014). This is despite of being exposed to various mathematics activities, which only shows that students need a unique turning point in their mathematics experience to engender a change of impression (Sanacore, 2008). Significantly, considerations of the effects of prior encounters with mathematics, embedded in social and cultural learning experiences are investigated and responded with appropriate adjustments since these may either assist the learning process or present difficulties (Evans, 2000).

Many students carry with them prior experiences of disaffection and low attainment that are transformed in their mathematics college experience which focus on highlighting mathematics application rather than knowledge acquisition and presenting a new image of mathematics as a useful 'tool for life' (Dalby, 2014). Also, it was found that students actively participate in mathematics learning with consistent interaction, clear and explicit explanation, discussion, and negotiation (Ewing, 2009).

Students' Mathematics Learning Strategies

Varied learning strategies that individuals used to process and solve mathematical problems often determine the success and degree of learning one gets (Anthony 1994, Thiessen 2008, Gasco 2013, Magen-Nagar 2016). Learning strategies can be within the realm of simple memorization to sophisticated strategies that individuals use for mathematics (Schoenfeld, 1992).

Pintrich is a key author in the study of learning strategies largely for the proposed *Motivated Learning Strategies Questionnaire (MLSQ)* (Pintrich, Smith, Garcia, McKeachie 1991), which had been widely used in educational researches. The questionnaire included three strategies categories: cognitive strategies, which includes rehearsal, organization, elaboration and critical thinking; metacognitive strategies, which includes planning, monitoring and regulation; and resource management strategies, which includes time and study environment management, peer learning and help seeking. In the PISA study (OECD 2003) there were three learning strategies that were examined: memorization strategies, which includes basically memorizing everything; elaboration strategies, which includes relating the new concept to previously learned knowledge and looking for other study materials to deepen and further understanding; and control strategies, which included self-clarification of concepts and skills and making sense of the concept. Results of different studies done in mathematics learning strategies show that overemphasis on memorization strategies does not contribute to effective mathematics learning (Anthony 1994; Geary 2005; Thiessen 2008; Gasco 2013; Magen-Nagar 2016). Several researches on students' mathematics learning strategies suggest that the mathematical achievements of students improved more when using elaboration strategies intensively (Baroody 2006; Magen-Nagar 2016; Gasco 2013).

Relationship of Students' Mathematics Conception, Attitude and Achievement

The literature is rich in results that revealed a strong positive correlation between students' attitude and their academic performance (Mensah & Kurancie, 2013; TIMSS 2017, Ngussa and Mbuti, 2017; Dagnew, 2017). This only shows that the attitude of students towards a subject is an important factor that determines better academic performance.

Also, it has been concluded from many recent studies that constructivism has positive effects on students' learning and has great potential in improving students' attitude towards learning as presented in meta-analyses done in the effects of constructivism (Ayaz, M. & Sekerci, H., 2015; Toraman, C. & Demir, E., 2016). This resulted in more acceptance and use of this learning methodology which posits that learning is a process of establishing a link between the new information and the information that exists in individuals. Furthermore, it argues that students are not the mere receiver of information but also creators of their understanding and knowledge.

Gaps in Literature

While an absolutist tends to focus on the acquisition of skills or discrete pieces of mathematical knowledge, constructivist reformers generally focus on the processes by which students create a conceptual understanding of classroom mathematics. This strong and extreme polarization of views of mathematics had deterred the maximum growth of mathematics education (Stemhagen, 2004). Absolutism strongly posits that knowledge and truths cannot be created but only discovered which ignores the cognitive capacities of the individual thinker and the social and historical facets of knowledge. Strands of absolutism such as formalism and progressive absolutism tend to turn to subjective knowledge when faced with the philosophical foundation of the truths of their way of thinking. However, a strong version of constructivism recognized the social and environmental influences in the way humans create their knowledge and strongly proposed subjective knowledge putting the individual thinker at the center of the whole knowledge creation had rejected the absolute stability of mathematics.

The contextualized approach established in this study considers the unique stability of mathematical knowledge while recognizing the ability of the thinking subject to construct understanding which may result in the growth of knowledge. The Contextualized Absolutist-Constructivist perspective of mathematics posits that mathematical knowledge is not discarded in the face of new concepts. It argues that the specificity of the domain validity of the existing mathematical knowledge is expanded to welcome the truth of the newly discovered or created mathematical realms. As a consequence of this contextualized perspective to mathematics, the teaching of mathematics is suggested to take a paradigm shift of implying mathematics as partly found and partly made by allowing students to practice mathematical skills repeatedly and apply such in solving practical problems. This contextualized perspective suggests that mathematics teachers practice absolutist perspective in teaching mathematical skills through demonstrations, repetitions, and drills until such time that understanding and mastery of these concepts and skills are attained by students. After which, teachers should engage students in practical problem-solving activities which would prompt students to realize the connections of the mathematics that they have learned, their lives, and the world they live in.

However, it is also suggested that if students show evident mastery of skills and strong background knowledge of the concept, they should be engaged in worthwhile mathematics application activities. Moreover, in this contextualized perspective, it is highly suggested that teachers encourage students to apply what they have learned to the things that interest them and see what knowledge or relation they can create or discover which may stimulate challenge and engender motivation. In line with this, the researcher realized after cautiously looking at the roots, causes, and stands of the two parties in the math arena, that when concepts that are objective and logical are matched to the student's thinking capacity can prompt the generative individual to make meaning of it based on his own experience. In other words, the appropriateness of the mathematical activity to that of the capacity of the individual is the bridge to unify the polarization of views in mathematics. This is undergirded by Bruner's Guided Discovery (1961) which suggest that learners build on past experiences and knowledge, use their intuition, imagination and creativity, and make meaning of their experience to learn.

This consequently suggest classroom implications to better mathematics education. The Contextualized Absolutist-Constructivist perspective of mathematics and its consequence mathematics teaching implications aim to improve students' mathematics conception, attitude, learning experiences, and achievement.

Conceptual Framework

The study was guided by the Conceptual Framework illustrated below in Figure 2.1.

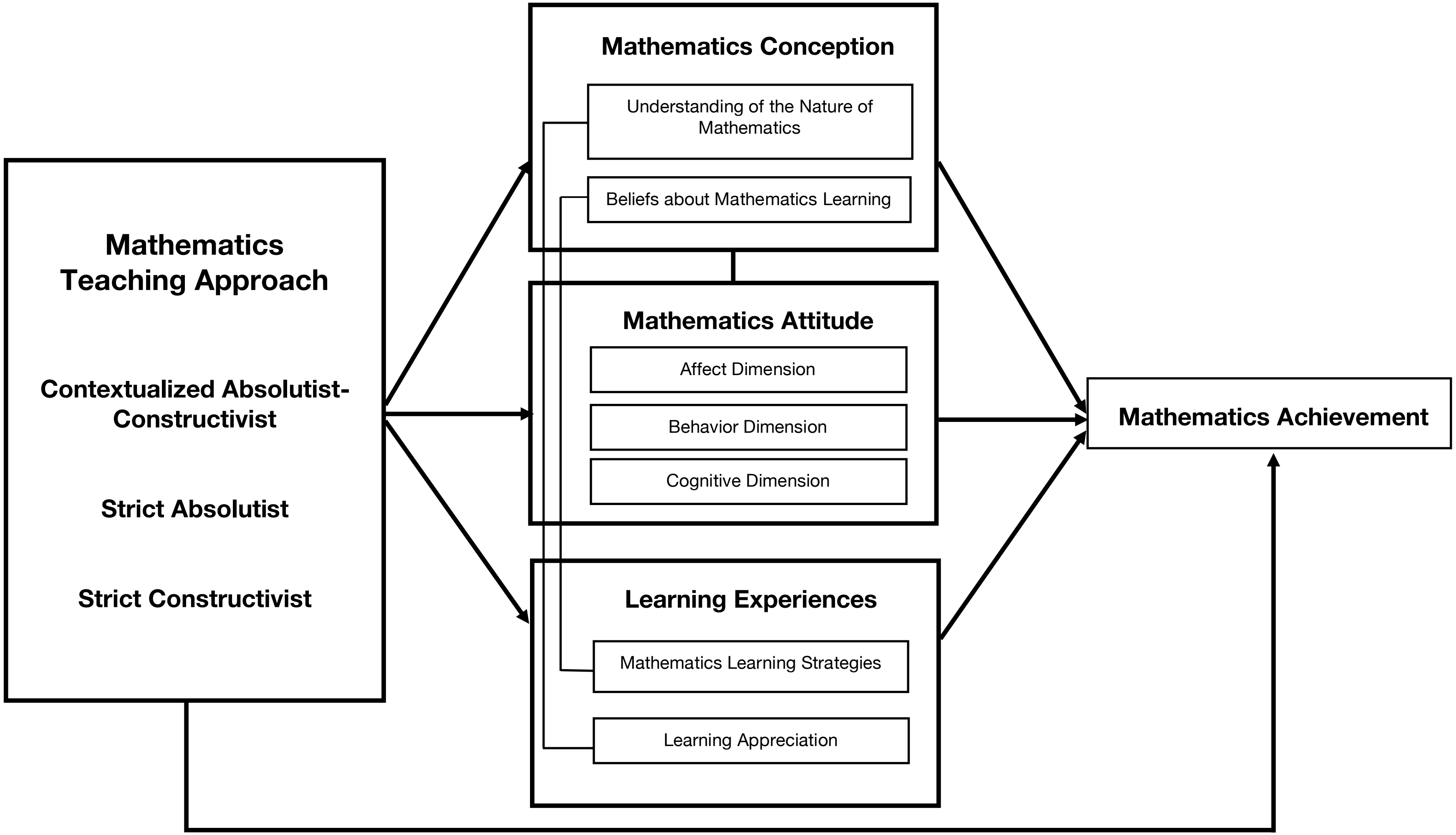

Figure 2.1. Conceptual Framework of the Study

The study made a comparative analysis of the effects of the three teaching approach: Strict Absolutist or the conventional approach of lecture-discussion method, Strict Constructivist, and Contextualized Absolutist-Constructivist teaching approaches to students' conception, attitude, learning experiences and achievement in mathematics.

The clashing views of mathematics that caused problematic issues in teaching and learning of the discipline encouraged the objective of this study to offer an alternative perspective that mends that unsettling clash in the mathematics arena. The Contextualized Absolutist-Constructivist perspective suggested in this study was established through scrutiny and comparisons of the opposing views in mathematics and their consequent mathematics teaching implications. This alternative perspective to mathematics suggests an understanding of mathematics that accounts for its unique stability and remarkable universality yet still recognizes its ability to change, improve and grow. Classroom implications of the Contextualized Absolutist-Constructivist perspective were realized through actual classroom teaching.

The strict absolutist mathematics teachers (traditionalists) – those who call for more rigor and a "back-to-basics" approach to mathematics – view mathematics as a given and ready-made truth to be passed on to students. This approach employs the prevalent traditional lecture-discussion method where students redo what the teacher had demonstrated. On the other hand, the strict constructivists – those advocating a more student-centered and applied approach to mathematics education – view individuals as the generators of knowledge. This strict polarization of views made educators and practitioners choose which side to take, adapt, and put into practice. It has been evident in the past that the problem with the nature of mathematics and mathematics education when put into practice will lead to an inevitable fall between these two camps of mathematics philosophy: absolutism or fallibilism (Ernest, 1991). This study aims to suggest that one has to realize that if he picks up one end of the stick, he also in one way or another had picked up the other end. The Contextualized Absolutist-Constructivist perspective to mathematics suggested in this study argues that it could be more useful to think of mathematics as partly found and partly made. Thus, a proposition for a more harmonic interaction of these philosophies is suggested which in turn can offer worthwhile classroom implications to improve mathematics learning.

Based on the extensive literature review done, it was found out that constructivist ideas of learning can provide useful mathematics practices that result in more positive effects to students' academic performance and attitude toward learning when compared with the traditional lecture-discussion method or the absolutist approach (Ayaz, M. & Sekerci, H., 2015; Toraman, C. & Demir, E., 2016). In the constructivist approach, students are more involved and responsible in their learning rather than staying as passive recipients of information (Ackermann, 2001). In this study, the constructivist approach vis-à vis the traditional strategy to teaching were compared with the Contextualized Absolutist-Constructivist teaching approach to determine which of the three pedagogical approaches fostered a better understanding of the nature and purpose of mathematics, promote a more positive attitude towards learning, and provide more meaningful learning experiences to students.

This study also looked into the effects of the three pedagogical approaches discussed above on students' mathematics conception. The student's understanding of the nature of mathematics was used as a sub-variable to further delve deeply into how students view and understand the components and totality of the mathematics that they do. This aimed to determine the level of students' understanding of the nature, origin, history, purpose, and use of mathematics which are integral in learning and appreciating the discipline (Presmeg, 2002; Fielding, Fuller, & Loose, 1999; Rudduck & Flutter, 2000; Young-Loveridge, 2005; Young-Loveridge & Taylor, 2005; Young-Loveridge, Taylor, & Hawera, 2005). It was further emphasized by Swetz (1994) that "The history of mathematics supplies human roots to the subject. It associates mathematics with people and their needs." This only concreted the significance of using the history of mathematics in imparting understanding of the nature of mathematics by humanizing the discipline. However, in the actual mathematics classroom this is commonly overlooked and not given attention. As Whitrow (1932) reported it, "Unfortunately, the historical dimension of mathematics is deficient, ignored or viewed as an "exotic luxury". Moreover, research on the inclusion of historical account of mathematics in the teaching of mathematics is scarce (Panasuk, R. M. & Horton, L.B., 2012). Hence, in this study the teaching of the nature of mathematics is done through the use of historical account of mathematics and its applications.

Students' beliefs about learning mathematics were also used as a sub-variable of mathematics conception to further analyze students' view of themselves as learners of mathematics. This construct includes students' perception of how learning mathematics can create new mathematical knowledge and how this knowledge relates to the world.

Moreover, this study used the Affect-Behavior-Cognition (ABC) Attitude model (Azjen, 1993) as a guide in comprising the components that influence students' attitudes towards mathematics. In this study, attitude was defined as one's beliefs and knowledge of his competencies and abilities in mathematics (affect); inclination to act towards mathematics (behavior); and positioning of oneself as a user of mathematics (cognition). These sub-variables were studied to shed more light on students' attitudes towards mathematics and its inevitable and critical effects on students' academic achievement.

The study also determined the effects of the three teaching approaches on students' learning experiences. Learning Experiences refers to the activities, strategies, and the general overall account of what students experienced in their mathematics class. This was measured by the Learning Experiences Inventory (LEI) which was developed for this study. Students' learning appreciation served as a sub-variable to analyze closely how students perceived their mathematics classroom activities and their possible effects on their learning. Learning appreciation construct pertains to students' account and regard on their learning experiences; of whether they like or dislike what they had experienced during their classroom engagements. This includes students' perception of whether the learning experience is enjoyable, interesting, and motivating. Furthermore, students' learning strategies also served as a sub-variable under learning experiences. This was done to determine how students cope and adjust to learning mathematics concepts. This shed more light on the different learning devices students used in dealing with mathematics which can provide a better understanding of effective mechanisms and strategies that can systematically help students in their mathematics learning.

This study also aimed to determine if there was a meaningful relationship between students' mathematics conception, attitude, and learning experiences and if these factors positively predict better mathematics achievement.

Hypotheses of the Study

The research hypotheses of the study are:

1. The mathematics conception, attitude, learning experiences, and achievement posttest mean scores of the students in the Contextualized Absolutist-Constructivist approach group is significantly higher than that of the Strict Absolutist (traditional) and Strict Constructivist teaching approach groups.

2. The mathematics conception mean gain score of students in the Contextualized Absolutist-Constructivist teaching approach group is significantly higher than that of students in the Strict Absolutist and Strict Constructivist teaching approach groups in terms of:

    a. understanding of the nature of mathematics; and

    b. beliefs about mathematics learning.

3. The mathematics attitude mean gain score of students in the Contextualized Absolutist-Constructivist teaching approach group is significantly higher than that of students in the Strict Absolutist and Strict Constructivist teaching approach groups in terms of:

    a. affect dimension;

    b. behavior dimension; and

    c. cognitive dimension.

4. There is a significant positive relationship between students'

    a. mathematics conception and attitude;

    b. understanding of the nature of mathematics and learning appreciation; and

    c. beliefs about mathematics learning and mathematics learning strategies;

5. Students' mathematics conception, attitude, and learning experiences are significant positive predictors of mathematics achievement.

Definition of Terms

The following terms are defined as used in the study.

Contextualized Absolutist-Constructivist Perspective (AC) is a contextualized perspective to mathematics that recognize the significance of relating mathematical concepts to practical, real-world context. This contextualized perspective was established as an adequate view of mathematics through the review and utilization of various absolutist and constructivist philosophies and ideas. This contextualized absolutist-constructivist view suggests that mathematics is a stable body of knowledge that is also capable to expand and grow as a result of the unending human exploration and use of mathematics. It further argues that existing mathematical knowledge is not falsified or discarded in the face of newly created or discovered information but rather its scope validity is changed and enlarged for the truth of a new realm. This is a perspective that melds opposing polarized absolutist and constructivist philosophical views in mathematics.

Contextualized Absolutist-Constructivist Teaching Approach is a suggested approach to teaching mathematics which emphasizes the significance of explicitly teaching ideas of the nature of mathematics and its historical account, allowing students to be curious and creative in mathematics by encouraging them to find varied ways of solving problems, and matching the content to the background knowledge of the students.

Affect Attitude Dimension pertains to students' beliefs and knowledge of their abilities in mathematics (Azjen, 1993). This also includes students' emotions and feelings in learning mathematics.

Behavior Attitude Dimension pertains to students' inclination to act towards the subject (Azjen, 1993). This would also include students' motivation as reflected in their actions, commitment, and achievement in mathematics.

Cognitive Attitude Dimension pertains to students' positioning of themselves as a user of mathematics (Azjen, 1993). This also pertains to how students perceive the usefulness and relevance of mathematics.

Learning Appreciation pertains to students' account and regard on their learning experiences; of whether they like or dislike what they had experienced during their classroom engagements. This includes students' perception of whether the learning experience is enjoyable, interesting, and motivating.

Learning Experiences refers to the activities, strategies, and the general overall account of what students experienced in their mathematics class. This was measured by the Learning Experiences Inventory (LEI) which was developed for this study. Further data of students' learning experiences were obtained from students' learning journals, reflection exit cards, and interviews.

Mathematics Attitude pertains to the predisposition and impression of students towards mathematics. The study made use of the ABC Attitude Model (Azjen, 1993) in measuring students' attitudes towards mathematics. This model has three dimensions namely: affect, behavior, and cognition. This was measured using the attitudinal questionnaire (Student Mathematics Attitudinal Questionnaire – SMAQ) developed and validated for this study.

Mathematics Conception pertains to students' general knowledge of what mathematics is all about. This also includes students' beliefs of how they can learn and use mathematics. This was measured using the Student Mathematics Conception Questionnaire (SMCQ) developed and validated for this study.

Understanding of the Nature of Mathematics pertains to students' understanding of the history, genesis, and role of mathematics. This also includes students' understanding of whether mathematics is an absolute static truth or a dynamic discipline capable of expanding and growing. This was measured using the developed and validated Student Mathematics Conception Questionnaire (SMCQ).

Strictly Absolutist Teaching Approach refers to the method of teaching adhering to strict lecture and demonstration of absolute and concrete knowledge of mathematics which recognizes a unique solution or method to mathematics. This is similar to the lecture-discussion method prevalent in the teaching of mathematics which expects students to repeat what the teacher had shown.

Strictly Constructivist Teaching Approach is the teaching method that adheres to the belief that learners create their knowledge and social interaction is key to learning. This teaching method will be performed using student groupings and activities that will allow students to create and structure their knowledge in mathematics having minimal guidance and instruction from the teacher.

Students' Beliefs about Mathematics Learning pertains to students' general idea of how one learns and uses mathematics. This includes students' perception of how learning mathematics can create new mathematical knowledge and how this knowledge relates to the world.

Students' Mathematics Learning Strategies refers to the different learning devices and mechanisms students used to cope in and learn mathematics.

Students' Mathematics Achievement pertains to students' mathematics measure of learning and achievement. This was measured using students' Mathematics Achievement Exam scores.

Chapter III

METHODOLOGY

This chapter includes the discussion on the research design, the participants of the study, the research instruments, the data collection procedure and the data analysis procedure.

Research Design

The quasi-experimental method of research particularly the pretest-posttest research design was used for the conduct of the comparative analysis on the influence of Contextualized Absolutist-Constructivist teaching approach, as well as Strict Absolutist and Strict Constructivist teaching approaches on students' mathematics conception, attitude, learning experiences and achievement.

$$\begin{array}{ccc} O_1 & X_1 & O_1' \\ \hline O_2 & X_2 & O_2' \\ \hline O_3 & X_3 & O_3' \end{array}$$

Figure 3.1. Research Design

Where
O_1, O_2, O_3– pretests
O_1', O_2', O_3'– posttests
X_1 – teaching approach (Strict Absolutist or conventional approach)
X_2 – teaching approach (Strict Constructivist approach)
X_3 – teaching approach (Contextualized Absolutist-Constructivist approach)

The design is considered as one of the most widely used designs in educational research since classes are intact groups and randomization of sample is difficult to conduct. Consisting of three groups namely the Contextualized Absolutist-Constructivist, the Strict Absolutist, and the Strict Constructivist teaching approach groups, the design gave pretest and posttest to each group. The controlled group was assigned as the Strict Absolutist approach or the traditional-lecture discussion method of teaching. The broken line between the groups suggested that there has been no randomization done. This design is commonly used with groups or participants that are naturally assembled such as those in classrooms. The assumption is that the groups are equal but, in case there are effects of extraneous variables identified, the Analysis of Covariance (ANCOVA) was used. An advantage of this design is that since classes are chosen “as is”, possible effects of reactive arrangements can be minimized.

The Sample

This study involved 112 college freshmen students of Bulacan State University (BulSU)-Main Campus in Malolos City, Bulacan who were enrolled during the Second Semester of the Academic Year 2019-2020. Purposive sampling was used to determine the three intact classes that became part of the study. The three classes were all freshmen Bachelor of Science in Mathematics students majoring in Applied Statistics (AS) and Computer Science (CS). The BS Math AS class consisted of thirty-nine (39) students while the BS Math CS 1B and 1D classes consisted of thirty-five (35) and thirty-eight (38) students, respectively. All the three classes were scheduled in the morning (see Appendix A: Class Schedule). The teacher-researcher taught the course “Fundamental Concepts of Mathematical Structure,” covering the topics from the start of the semester to the midterm examination coverage. This included the following unit topics: Mathematical Logic, Theory of Sets, and Introduction to Number Theory (see Appendix B: Course Syllabus). The teaching approach assignment to each class was determined using random sampling, particularly the lottery method. The BS Math AS 1A class was assigned to Strictly Constructivist approach, while the BS Math CS 1B and CS 1D were determined as the Strictly Absolutist (traditional lecture-discussion method) class and Contextualized Absolutist-Constructivist teaching approach class, respectively.

Eight (8) lesson plans (Propositions, Tautologies and Logical Equivalence, Arguments and Proofs, Sets and Operations, Set Identities and Laws of Algebra of Sets, Divisibility and Euclidean Algorithm) for the three (3) lesson units (Logic, Sets and Divisibility) were prepared for each teaching approach to ensure accurate and credible classroom teaching implementation (see Appendix C: Sample Lessons). These lesson plans were checked and validated by two (2) mathematics experts and teachers handling the course Fundamental Concepts of Mathematical Structure (FCM). The two mathematics experts were the Program Chair and Mathematics Department Chair of the College of Science in BulSU.

The experimentation period was conducted from the start of the First Semester of the Academic Year 2019-2020 until the midterm examination period, hence taking up all midterm topics. This was approximately 9 weeks or 27 contact hours or half of the semester.

Classroom observations in the three experimental groups were conducted by the College of Science's Dean, Department Head and Program Chair to ensure that the teaching experimentation was done appropriately. The class observers were given an evaluation form (see Appendix D: Classroom Observation Form) to assess and comment on the classroom implementation of the three teaching approaches.

The researcher sought permission and approval on the conduct of the study from the Vice President for Academic Affairs and the University President (see Appendix E: Permission Letter). Also, prior coordination with the Dean of the College of Science and the Head of the Mathematics Department was done.

The Instruments

Four (4) research instruments were developed and validated: Students' Mathematics Conception Questionnaire (SMCQ), Mathematics Learning Experience Questionnaire (MLEQ), Students' Mathematics Attitudinal Questionnaire (SMAQ) and Mathematics Achievement Test.

The Students' Mathematics Conception Questionnaire (SMCQ) is a 26-item questionnaire of student's mathematics conception that generally aims to analyze student's understanding of the nature of mathematical knowledge and general idea of the practical applications of the discipline. This questionnaire also aims to determine how students view themselves as learners and users of mathematics. The questionnaire also determines the level of students' knowledge of their competencies and abilities in mathematics.

The Mathematics Learning Experience Questionnaire (MLEQ) is comprised of thirty (30) items that aims to understand the different learning experiences of students in the three comparative groups. This was further aimed to provide quantitative data to establish the information from students' answers in their learning journal and reflection exit cards which are given at the end of every lesson to assess students' learning experiences.

To measure the attitude of the students towards Mathematics, a Mathematics Attitudinal Questionnaire (SMAQ) was developed that measured three (3) dimensions of attitude (affect, behavior and cognitive) as suggested in the ABC Attitude Model of Azjen (1993). The affective component is the emotional response (liking/disliking) towards an attitude object which in this case is mathematics. The behavioral component is a verbal or overt (nonverbal) behavioral tendency by an individual and it consists of actions or observable responses towards the attitude object (Wicker, 1969). The cognitive component is an evaluation of the entity that constitutes an individual's opinion (belief/disbelief) about mathematics. There were positive and negative statements in the questionnaire to validate the consistency of the respondents. For positive statements, the following scoring scheme was used: Strongly Agree – 6 points; Moderately Agree – 5 points; Slightly Agree – 4; Slightly Disagree – 3; Moderately Disagree – 2; and Strongly Agree – 1. On the other hand, the scoring was reversed for the negative statements. The final form of the attitudinal questionnaire was comprised of 15-item questions, five (5) for each of the three (3) dimensions.

The rating scale of the questionnaires was interpreted as follows: 6 – Strongly Agree, 5 – Moderately Agree, 4 – Slightly Agree, 3 – Slightly Disagree, 2 – Moderately Disagree and 1 – Strongly Disagree. The questionnaires development included the following phases: planning the test, content validation, pilot testing and item analysis, and evaluation of the test. The first draft of the questionnaires were content validated by four (4) doctorate degree holder in mathematics: 1 from Bulacan State University who is the Program Chair of the BS Mathematics program under the College of Science; 2 from Philippine Normal University who one of them is the dean of the College of Science and another is a faculty member teaching mathematics; and 1 from the University of the Philippines teaching mathematics. They evaluated each item on its relevance, importance and clarity (see Appendix F: Expert Instrument Evaluation Form).

The questionnaires were pilot tested to 120 college students taking up BS Mathematics in the College of Science of Bulacan State University**.** The construct reliability or internal consistency of the items was assessed using the Cronbach alpha of each subscale and the instrument itself. Both the measures used in this instrument displayed excellent internal consistency exceeding the reliability estimates ($\alpha = .70$). These results and the item-correlation values were used in finalizing the items in the instrument. The item-correlations values were examined to analyze internal consistency reliability. These values provided assessment of item redundancy or the extent to which items are assessing the same content.

The Cronbach's alpha reliability coefficients for each questionnaire was obtained to ensure internal consistency. Table 3.1 presents the results of this analysis.

Table 3.1
Cronbach's alpha Reliability Coefficients

Questionnaire	No. of Items	Alpha
SMCQ	26	.869
MLEQ	30	.831
SMAQ	30	.849

As revealed by the alpha coefficients in Table 3.1, all values exceed the recommended reliability alpha estimates of $\alpha = .70$ (Nunnally, 1967) thereby validating that there is excellent internal consistency in the three Likert scale questionnaires developed: Students' Mathematics Conception Questionnaire, Mathematics Learning Experience Questionnaire, and Students' Mathematics Attitudinal Questionnaire (see Appendix G: Questionnaires - Final Form).

Among the three Likert scale instruments presented above, the SMCQ (M = 4.95) obtained the highest mean rating from students who evaluated the instruments (MLEQ M = 4.40, SMAQ M = 4.46). This can be accounted for by the fact that the students are Third-Year BS Mathematics students who had taken up higher mathematics courses thus exposing them to wider and deeper idea of mathematics and its use.

An achievement test in mathematics was developed to measure the level of knowledge students have about the following topics in the course Fundamental Concepts of Mathematical Structures: Propositions, Tautologies and Logical Equivalence, Arguments and Proofs, Sets and Operations, Set Identities and Laws of Algebra of Sets, Divisibility and Euclidean Algorithm. A Table of Specification (TOS) was first structured to ensure that the items in the test are spread appropriately among Bloom's Taxonomy, clarify learning outcomes and match the methods of instruction with assessment (see Appendix H: TOS). These items are of varying degree of difficulty to ensure students' understanding and learning of the mathematical concepts. The test was a multiple-choice type of examination with four options. The multiple choices were carefully made to avoid leading the students to the correct answer, thereby resulting to weak assessment data. Each option was made by predicting students' possible solution or method resulting to values or answers that are incorrect.

To validate the content of the achievement test, the four (4) experts in mathematics who also evaluated the Likert scale instruments in this study were asked to meticulously check each item by confirming the correctness of the questions and answers in the test. As a result of this validation procedure, 2 (two) questions were omitted and replaced, and 3 (three) were reworded. The mathematics achievement test was pilot tested to 120 Third-Year BS Mathematics students of Bulacan State University who had taken the course Fundamental Concepts of Mathematical Structures. The data gathered from this achievement pilot testing was used to see relationship between student's test score and their FCM course grade using Pearson Product-Moment Correlation. This is done to establish the validity of the achievement test. The correlation coefficient was found to be -0.630. This value coefficient is negative for the reason that the grading system in the university has 1.00 as the highest and 5.00 as the lowest. The computed correlation coefficient indicates a high correlation between students' mathematics achievement test score and their FCM course grade. Hence, the final form of the achievement test (see Appendix I: Mathematics Achievement Test Final Form) is valid and appropriate to use. To further determine the test reliability, the Kuder-Richardson (KR) Formula 20 was used. The KR-20 was used because the items in the test vary in degree of difficulty and there are dichotomous choices (right or wrong). It also was also used as the analysis tool that checked the internal consistency of the test. The KR-20 computed value for the mathematics achievement test was .768 indicating high reliability of the items and the whole instrument itself (see Appendix J: Kuder-Richardson Formula 20 Reliability Coefficient).

The mathematics achievement test was used as pretest in the study. However, due to the sudden class suspension and community lockdowns caused by the COVID-19 pandemic the mathematics achievement posttest was not administered. As a result, the quiz items which were matched to the items in the mathematics achievement test based on the concept or skill it measure were used as posttest instead. There is no validation done for the quiz items.

Contextualized Absolutist-Constructivist Teaching Approach

As a consequence of the Contextualized Absolutist-Constructivist perspective to mathematics, this study endeavors to suggest a classroom approach which presents the melding of the two polarized perspectives in mathematics. In light of mathematics education, the Contextualized teaching approach values appropriate matching of content and the individual’s capacity for a meaningful interaction and construction of understandings. The key feature of this alternative approach is the recognition of the interaction between what is given and those who construct mathematical understandings. The Contextualized approach to mathematics teaching suggests that teachers can recognize and respond to students’ prior knowledge or entry points by tailoring classroom discussion to where the students are at a particular lesson.

Lev Vygotsky, a Russian psychologist, proved that individuals learn best in accordance with their readiness (Allan & Tomlinson, 2000). That is, the difficulty of skills taught should be slightly in advance of the child's current level of mastery. This is grounded in Vygotsky’s work and the zone of proximal development (ZPD), the range at which learning takes place. Psychologists tell us that a student learns only when a task is a little too hard for the student. When a student can do work with little effort the student is not learning, but rather rehearsing. When a student finds a task beyond his reach, frustration, not learning, is the result. Only when a task is a bit beyond the student's comfort level, and the student finds a support system to bridge the gap, does learning occur. This theoretical influence provides a concrete foundation for the contextualized approach. In this approach, the inevitable “some” way of transmission that takes place as certain mathematical concepts and truths are defined and elaborated to students is recognized. Moreover, in this

Contextualized Absolutist-Constructivist teaching approach, individualized mathematical solutions and methods are encouraged. It suggests that teachers may present mathematical concepts and allow students to practice mathematical skills in absolutist approach which can be done through demonstration, repetition and drill. This is done until such time that students gain understanding and mastery of the mathematical concepts and skills. When students attained mastery of mathematical skills and concepts, students are given meaningful bona fide problem-solving activities that are within the interest and passion of the students. In this way, students realized that mathematics is something that they need, can do and create. This is hoped to foster deeper understanding of concepts, inculcate appreciation of the use and engender mathematics discovery or creation.

Furthermore, this Contextualized approach to mathematics teaching suggests that teachers can incorporate rich historical account of mathematical concepts to each lesson whenever possible so as to provide students the necessary background, understanding and appreciation of the discipline. This will provide students better understanding of the nature and purpose of mathematics. These meaningful additions are believed to foster increased mathematics interest and curiosity which in turn can result to increased mathematics engagement and learning.

The researcher taught the three teaching approach groups: Strict Absolutist, Strict Constructivist and the Contextualized Absolutist-Constructivist teaching approach groups. The class session for each teaching approach was once a week with three (3) hours per session which lasted for nine (9) weeks or half of the semester.

Table 3.2

Lesson Summary Comparisons of the Strict Absolutist, Strict Constructivist and Contextualized Teaching Approaches

Lesson Stage	Aspect	Strict Absolutist	Strict Constructivist	Contextualized
Pre-lesson Preparation	Responding to Students' Individual Learning Needs	In the Strict Absolutist teaching approach, the mathematics classroom practice is a one-size-fits-all.	In the Strict Constructivist teaching approach students adjust their knowledge structures to the given mathematical problem.	In the Contextualized teaching approach students' prior knowledge and entry point to a particular lesson are being recognized and responded to by tailoring classroom activities according to their needs. Students' prior knowledge is determined using a Reflection Exit Card. Teachers values appropriate matching of content and the individual's capacity for a meaningful interaction and construction of understandings.

Lesson Proper	Mathematical Concept and Nature of Mathematics Teaching	Students are taught using the traditional lecture-discussion method where students listen passively to the teacher.	Students are given pre-prepared worksheets with detailed instructions for students to follow leading them to exploring the mathematical concept on their own.	The teacher conducts direct instruction allowing students to gain understanding of the mathematical concept. The contextualized approach acknowledges the objective formal mathematics which are used for mathematical intuition and thinking processes.
		Nature of mathematics understanding and history of mathematics are hardly imparted.	Implied exposing of students to the nature of mathematics.	Explicit teaching of the nature of mathematics and rich incorporation of mathematics historical account are highlighted. ***Teaching strategy used:*** Historical account of the concept. ***Frequency:*** Always. This strategy is always used in this Contextualized

				Absolutist-Constructivist teaching approach.
Lesson Activity	Mathematical Solution	Students are given drill exercises to practice what they have learned as how the teacher demonstrated it on the board.	Students are asked to formulate/construct the mathematical ideas and concepts based on the examples in the worksheet. (Students will work on this together with minimal guidance from the teacher. However, they are allowed to ask questions.)	The teacher uses flexible grouping, allowing students to group themselves as they find it effective. Students are given a worksheet they will work on together. ***Teaching strategy used:*** Real-life experiences. ***Frequency:*** Always. This worksheet includes real-life problems involving the mathematical concept they just learned. Questions are structured in such a way that students will see the relevance of what they are learning to that of the things outside their classroom.

		Mathematical solutions are constructed based on the teacher's demonstration.	Exploration and discovery of mathematical solutions are encouraged.	***Teaching strategy used:*** Individualized solutions. ***Frequency:*** Whenever possible. Individualized mathematical solutions and methods are encouraged.
Assignment	Relating mathematics to the world and showing the human influence to its development	Students are given set of mathematical problems on the concept they have learned.	Students are given a set of mathematical problems that are structured involving real-life scenarios.	Students are encouraged to apply the mathematical concept they have learned to their interests and passion. They are also challenged to be curious and creative in exploring possible applications of what they have learned. Students are also encouraged to use mathematical software and applications.
Assessment	Students are given a common quiz at the end of each lesson unit to assess their learning.			

It can be observed from Table 3.2 that in the Contextualized teaching approach, the students' prerequisite knowledge of the topic is assessed prior to the lesson through the use of a reflection exit card (see Appendix K: Sample Reflection Exit Card). Students accomplished this reflective assessment before each lesson ended.

The purpose of this reflection exit card is two-tiered. First, the set of questions in the reflection exit card assessed students' mastery and understanding of the topic taught. This provided the teacher-researcher the needed assurance that students understood the concept and gained the skills taught and were ready for the next lesson. Second, the reflection exit card was an opportunity for the teacher to ask questions relative to the next topic which will help in planning the classroom activities for the next lesson. Furthermore, this was done in order to respond appropriately to students' varying learning needs and entry points. The data gathered from these reflective assessments were used to plan, prepare and execute the next mathematics lesson that fit students' situation or context. Reflection-exit cards used on this study included questions about students' interest on a particular topic, firmly-held mathematics misconceptions or difficulty on a particular mathematical skill. Through this, the teacher-researcher determined, recognized and responded to students' entry point on each mathematics topic by designing classroom discussions which helped students in their relevance- and meaning-making learning processes. The Contextualized Absolutist-Constructivist teaching approach emphasizes the significance of relating students' prior knowledge and the concept to be learned. This is grounded in the works of Vygotsky' Zone of Proximal Development (1986) and Ausubel's (1982) Meaningful Learning Theory.

In the Strict Absolutist or the traditional teaching approach, quick assessment of prerequisite concepts is done as a class through recitation. However, this one-size-fits-all classroom activity does not provide meaningful individualized learning aid to students. The teacher-researcher discussed the mathematical concept and demonstrated the skill using a whiteboard while students listened. After each class discussion, the teacher provided the students a set of mathematics problems to answer. This is to assess students' mastery and understanding of what they have learned. The teacher occasionally asked the students to do board works where the students present their answers to the class.

In the Strict Constructivist teaching approach, students are grouped together allowing them to discover, develop and construct their own meaning-making. Students work on a learning task such as answering a set of problems, researching and reporting a mathematical concept history, decoding a mathematical pattern from a series of examples, among others. They were given minimal supervision and were encouraged to explore and be creative in their meaning-making and learning. It can be recognized that the implementation of the Strict Constructivist teaching approach in mathematics is different from how it is implemented in other fields such as in social sciences. This could be accounted for by the fact that mathematics is characterized as pure and hard (Biglan, 1973). It was found out that pure and hard subject matters tend to need more social connectedness and commitment compared to soft and applied disciplines. This explains the effectiveness of collaboration among students when dealing with mathematics tasks. Students can learn from their peers and relate efficiently their own understanding.

The lesson-discussion of absolute mathematics knowledge in the Contextualized Absolutist-Constructivist teaching approach is similar to that of the Strict Absolutist (traditional) approach where the teacher demonstrates the mathematical concept or skill and students are then given sets of drill exercises to assess and practice their understanding. However, in the Contextualized Absolutist-Constructivist teaching approach, the students are then given the chance to apply what they have learned to the things that interest them. Their interests are closely studied by the teacher through observations, interviews and students' answers on their reflection exit cards. This is argued to foster motivation and engagement among students which will consequently improve mathematics achievement. Students are also encouraged to collaborate and learn from each other which supported the social aspect of learning.

The Contextualized Absolutist-Constructivist approach to mathematics teaching argues that students' mathematics learning is more effective if they can see its relation to their lives and the things that interest them. This is viewed to foster more positive attitude and improved motivation in mathematics among students. Moreover, the Contextualized Absolutist-Constructivist approach to mathematics teaching suggests that aiding students to have meaningful connection between the concept learned and its application is significant in their meaning-making process. The students are encouraged to apply what they have learned to what interest them. These proactive steps to teaching mathematics bridges the mathematical concepts learned and the practical and functional use of it.

At the end of every lesson unit, students from all three teaching groups were given a quiz assessment which served as the measure of how well the students learn. Class observations and informal interview were also done to collect information from students' learning experiences.

Data Collection Procedure

At the start of the semester, the teacher-researcher administered the Student's Mathematics Conception Questionnaire (SMCQ), the Mathematics Attitudinal Questionnaire to students and the Mathematics Achievement Test. The student scores in these questionnaires were used as covariates in eliminating effects of any extraneous variable since the pretest results of the three groups are significantly different.

After the three approaches were used in the three class, the Student's Mathematics Conception Questionnaire (SMCQ), Mathematics Attitudinal Questionnaire, and Student's Learning Experiences Questionnaire were administered, again. The data collected served as posttest scores of the students in these instruments.

However, due to the COVID-19 pandemic, classes were suspended until the semester ended. The mode of learning abruptly shifted to online virtual classes which cut short the experimentation period. The supposed schedule of the university midterm week was the following week when the community lockdown and class suspension were implemented. As a result, the Mathematics Achievement Test was not administered to students which was supposed to serve as posttest scores of students to determine significant difference on the learning of the three experimental groups. As a result of this unfortunate event, the quizzes of the students were used as posttest scores. Each of the 40 item questions in the Mathematics Achievement Test was matched to an item in a quiz which measured the same mathematical concept or skill (see Appendix L: Mathematics Achievement Test and Quiz Items Matching).

Data Analysis Procedure

The Cronbach alpha reliability coefficient was used to assess the internal consistency of the developed Likert scale instruments for the study. On the other hand, the Kuder-Richardson Formula 20 was used to check the internal consistency of the mathematics achievement test because the items in the test vary in degree of difficulty and the test items has two dichotomous choices (right or wrong).

The mean and standard deviation were computed to describe students' mathematics conception, attitude and learning experiences. SPSS (Statistical Package for Social Science) was used in the data analysis, and significance level set at .05 for the tests of hypotheses.

One tailed t-test for dependent samples was used to determine if there is a significant difference between the pretest and posttest scores of the students in their Mathematics Conception, Mathematics Attitudinal, and Learning Experiences Questionnaires in each learning group.

To determine if there is a significant difference between the attitude of the students in the Strict Absolutist, Strict Constructivist and Contextualized Absolutist-Constructivist teaching approach groups, Analysis of Variance (ANOVA) was used. The same statistical test was used to determine if there is a significant difference between the Students' Mathematics Conception and Learning Experiences Questionnaire results in the three groups. Cohen's d or the standardized mean difference was used to determine the effect size. Furthermore, to determine which pairs among the teaching approach groups differ significantly as revealed by the results of ANOVA test, a post hoc test particularly the Tukey's Honest Significant Difference (HSD) was used.

The Analysis of Covariance (ANCOVA) was used for variables that showed inequality among groups prior to the experimentation or more particularly when pretest scores indicated significant difference among groups. The pretest score served as the covariate. ANCOVA analyzes the differences between the experimental and control groups on the dependent variable after taking into account any initial differences between groups on pretest measures or any other relevant independent variable. In this study, ANCOVA was performed in the Mathematics Achievement results.

The Spearman Rank-Order Correlation statistical test was used to determine if there is a significant relationship between students' mathematics conception and attitude. Spearman Rank-Order Correlation was used after the data set failed the assumption of linearity for Pearson's Correlation. The same statistical test was used to determine whether there exists a significant correlation between students' understanding of the nature of mathematics and their learning appreciation. The said correlation statistical tool was also used to test any significant positive relationship between students' beliefs about mathematics learning and their mathematics learning strategies.

To determine if mathematics conception, attitude and learning experiences are significant positive predictors of mathematics achievement, multiple regression was done. It is used to predict the value of the dependent variable (mathematics achievement) based on the value of two or more other variables (independent variables: mathematics conception, attitude and learning experiences).

Chapter IV

RESULTS AND DISCUSSION

This chapter presents the quantitative and qualitative analysis and interpretation of the data.

Students' Mathematics Conception

Table 4.1 presents the descriptive statistics for Students' Mathematics Conception pretest scores.

Table 4.1
Means and Standard Deviations for Students' Mathematics Conception Pretest Scores

			Mathematics Conception	
Teaching Approach Groups	*n*	*M*	*VI*	*SD*
Strictly Absolutist	35	5.04	High	0.55
Strictly Constructivist	39	4.93	High	0.43
Contextualized	38	4.90	High	0.43

n – Sample size *VI – Verbal Interpretation*
M – Mean *SD – Standard Deviation*

It can be seen from Table 4.1 that the 35 students in the Strictly Absolutist teaching approach group had the highest average mathematics conception score with 5.04 (*SD* = 0.55). This indicates that the students in the Strict Absolutist teaching approach group had better understanding of the nature of mathematics prior to experimentation. On the other hand, the students in the teaching approach groups that utilized the Strict Constructivist and the Contextualized Absolutist-Constructivist approaches had close means with 4.93 and 4.90 (*SD* = 0.43), respectively.

Table 4.2
One-Way Analysis of Variance of Students' Mathematics Conception Pretest Scores

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	.420	.210	.953	.389
Within Groups	109	24.004	.220		
Total	111	24.424			

Table 4.2 presents the results of the One-Way ANOVA done to the Mathematics Conception pretest scores of students. There is no statistically significant difference among the teaching approach groups in terms of the variable mathematics conception, $F(2,109) = .953$, $p = .389$ at .05 level of significance. The teaching approach groups were equal and comparable. Hence, any significant difference in the students' mathematics conception after giving the treatment can be attributed to the teaching approach implemented.

Table 4.3
Means and Standard Deviations for Students' Mathematics Conception Posttest Scores

		Mathematics Conception	
Teaching Approach Groups	*n*	*M*	*SD*
Strictly Absolutist	35	5.00	.49
Strictly Constructivist	39	5.21	.43
Contextualized	38	5.54	.34

Table 4.3 shows that the average Mathematics Conception posttest score of the students in the Strict Absolutist teaching approach group is 5.00 (*SD* = .49) and that of the Strict Constructivist teaching approach group is 5.21 (*SD* = .43). The students in the Contextualized Absolutist-Constructivist teaching approach group obtained the highest mean posttest Mathematics Conception score 5.54 (*SD* = .34).

Table 4.4
One-Way Analysis of Variance of Students' Mathematics Conception Posttest Mean Scores

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	5.38	2.69	14.82	000
Within Groups	109	19.79	.182		
Total	111	25.17			

Table 4.4, reveals statistically significant difference among the mathematics conception posttest mean scores of the students in the three teaching approach groups, *F* = (2,109) = 14.82, *p = 000*. This reveals that at least two groups differ statistically significantly in terms of mathematics conception. Moreover, the effect size ($\eta^2 = 0.21$) indicates a large effect based on Cohen and Miles & Shevlin (2001). This value reveals that 21% of the variance in mathematics conception can be explained by the teaching approach used.

Table 4.5
Tukey HSD Post Hoc Analyses Result of Mathematics Conception Posttest Scores

(I) Teaching Approach	(J) Teaching Approach	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	-.22	.099	.082	-.45	.02
	Contextualized	-.54*	.099	.000	-.78	-.30
Constructivist	Contextualized	-.32*	.097	.003	-.55	-.09

*. The mean difference is significant at the 0.05 level.

From Table 4.5, it can be seen that the posttest mean scores of the students in the Contextualized Absolutist-Constructivist teaching approach group is significantly higher than that of the students in both the Strict Absolutist (*MD* = -.54) and Strict Constructivist (*MD* = -.32) teaching approach groups, *p* = .000 and *p* =

.003, respectively. This indicates that after the teaching approaches implementation, students in the Contextualized Absolutist-Constructivist teaching approach group had better understanding of the nature of mathematics thus improving their perception of the discipline and beliefs in learning it.

Recent researches have focused in analyzing the importance of knowing students' varying and unique perspectives of mathematics in leading them to learning and appreciating the discipline (Fielding, Fuller, & Loose, 1999; Rudduck & Flutter, 2000; Young-Loveridge, 2005; Young-Loveridge & Taylor, 2005; Young-Loveridge, Taylor, & Hawera, 2005). In the Contextualized Absolutist-Constructivist teaching approach group, students were provided with rich learning materials that includes the history, genesis and developmental phases of the mathematics concept they are dealing with.

Also, in the Contextualized Absolutist-Constructivist teaching approach group students find mathematics tasks just a bit challenging and within their Proximal Development Zone (Vygotsky, 1981). Vygotsky argued that individual learns best when the concept presented is slightly above the individual's knowledge and capacity because this would prompt the meaning-making processing of the concept. This idea is further supported by Ausubel (1982) who suggested the concept of Meaningful Learning Theory which argued that meaningful learning occur as new information is incorporated into prior knowledge.

Below are unedited responses of students in their Reflection Exit Cards which supports the claim that when students are provided with learning activities and materials that are within their ZPD and they find learning a bit challenging but meaningful, their learning is maximized. The following were representations of the common theme evident from the students' journal entries.

Student A: *"The lesson was interesting because there's a lot of learnings. It's also a bit challenging because you have to think logically. The topics discussed clearly and not that fast."*

Student C: *"The lessons were confusing at first, especially since there were a lot of concepts to absorb, yet I was able to comprehend it better through the examples given and also the activities."*

Student E: *"It was engaging and easy to follow. There are times when the lesson is quite confusing but at the end, I was still able to follow through."*

Student L: *"It was a wonderful experience! The activities were neither hard or complicated."*

The results being shown were undergirded by Dewey's pragmatic and psychological philosophy of mathematics which views the importance of bridging the gap between the child's interest and experiences and to allow this foundation to grow and expand towards further inquiries within and outside the discipline.

However, students' Mathematics Conception posttest scores in the Strict Absolutist and Strict Constructivist teaching approach groups were not significantly different, *p = .082.*

Students' Attitude Towards Mathematics

Table 4.6 shows the descriptive statistics of the pretest mean scores of the students. The pretest mean scores were analyzed to ensure that the three intact groups were equal at the start of the experiment so that any significant difference found in the posttest results are due to the treatment given.

Table 4.6
Means and Standard Deviations of Students' Attitude towards Mathematics Pretest Scores

Teaching Approach Groups	*n*	Attitude towards Mathematics	
		M	*SD*
Strictly Absolutist	35	4.41	.33
Strictly Constructivist	39	4.54	.36
Contextualized	38	4.40	.35

Table 4.6 presents the descriptive statistics of the pretest mean scores of the students among the three teaching approach groups in terms of their attitude towards Mathematics. The three teaching approach groups obtained close mean scores with Strict Absolutist, Strict Constructivist and the Contextualized Absolutist-Constructivist teaching approaches having mean scores of 4.41, 4.54 and 4.40, (SD = .33, .36, .35), respectively.

To ensure that there is no significant difference among the mean scores of the students in the three teaching approach groups at the start of the implementation of the teaching approaches, a One-Way ANOVA was performed. The result of this analysis is presented in Table 4.7.

Table 4.7
One-Way Analysis of Variance of Students' Pretest Scores in their Attitude towards Mathematics

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	.47	.23	1.95	.147
Within Groups	109	13.01	.19		
Total	111	13.48			

Table 4.7 shows the result of the One-Way ANOVA performed in the pretest mean scores of the students in the three teaching approach groups. The result

reveals that there is no statistical significant difference in the pretest mean scores of the students in the teaching approach groups, $F(2,109) = 1.95$, $p = .147$ at the level of significance .05. This informs that the three teaching approach groups are comparable at the start of the experiment in terms of their attitude towards Mathematics.

Table 4.8
Means and Standard Deviations of Students' Attitude towards Mathematics Posttest Scores

Teaching Approach Groups	*n*	Attitude towards Mathematics	
		M	*SD*
Strictly Absolutist	35	4.57	.30
Strictly Constructivist	39	4.89	.35
Contextualized	38	5.01	.29

Table 4.8 shows that the Contextualized Absolutist-Constructivist teaching approach group obtained the highest mean in terms of their attitude towards mathematics, with $M = 5.01$ ($SD = .29$). This indicates that the students in the Contextualized Absolutist-Constructivist teaching approach group had better attitude towards Mathematics after they were exposed to the Contextualized Absolutist-Constructivist teaching approach. This is further indicative that the Contextualized Absolutist-Constructivist teaching approach has the potential to help students improve their attitude towards mathematics and foster better appreciation of the discipline.

On the other hand, the students in the Strict Absolutist and Strict Constructivist teaching approach groups obtained close mean scores with $M = 4.57$ ($SD = .30$) and $M = 4.89$ ($SD = .35$), respectively.

Table 4.9
One-Way Analysis of Variance of Students' Attitude towards Mathematics Posttest Mean Scores

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	3.71	1.86	18.62	.000
Within Groups	109	11.07	.10		
Total	111	14.78			

The result of the One-Way ANOVA performed in the posttest mean scores of the students in the three teaching approach groups reveals that there is a statistically significant difference among the mean scores, *F* = (2,109) = 18.62, *p = 000*. This informs that at least two groups have significantly different attitude towards mathematics. Moreover, the effect size computed as eta squared ($\eta^2 = 0.25$) reveals that there is a large effect, particularly 25% of the variance in the posttest attitude mean scores is caused by the teaching approach used in each group.

Table 4.10
Tukey's HSD Post Hoc Analyses Result of Students' Attitude towards Mathematics Posttest Mean Scores

(I) Teaching Approach	(J) Teaching Approach	Mean Difference	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	-.32	.073	.000	-.49	-.14
	Contextualized	-.44	.074	.000	-.61	-.26
Constructivist	Contextualized	-.12	.071	.211	-.29	.05

*. The mean difference is significant at the 0.05 level.

Table 4.10 shows that the posttest mean scores of the students in the Strict Constructivist (*MD* = .32) and Contextualized Absolutist-Constructivist (*MD* = -.44) teaching approach groups are significantly higher than that of the posttest mean scores of the students in the Strict Absolutist teaching approach group, both with *p* = .000. This indicates that after the experiment the students in the Contextualized

Absolutist-Constructivist teaching approach group had significantly improved their attitude towards Mathematics compared to the students in the Strict Absolutist teaching approach group. This means that the Contextualized Absolutist-Constructivist approach to teaching mathematics can significantly improve students' attitude towards mathematics thereby fostering positive perception and behavior towards the discipline and possibly improving their mathematics achievement since research had shown that students' attitude towards mathematics is highly correlated with their mathematics achievement (Mensah & Kurancie, 2013; TIMSS 2017, Ngussa and Mbuti, 2017; Dagnew, 2017).

Below are some verbatim responses of students in their Reflection Exit Cards which show that the Contextualized Absolutist-Constructivist teaching approach provides students learning opportunities which are engaging and interesting by explicitly incorporating the teaching of the nature of mathematics and the historical account of mathematical knowledge they are learning. Presented below are representations of several similar responses of students.

Student E: *"It is engaging and enjoyable!"*

Student C: *"The activities were fun because we have time to apply and understand the lesson more."*

Student F: *"I find the lesson teaching specifically the methods of deduction so challenging. It's not about finding the answer with a given formula, it's about proving and I enjoy it.*

However, the posttest mean scores of the students in the Contextualized Absolutist-Constructivist teaching approach group (*MD* = .12) is found to be not significantly different from that of the posttest mean scores of the students in the Strict Constructivist group, *p* = .211 in terms of their attitude towards mathematics.

The results of the multiple comparison test done implies that both the Strict Constructivist and Contextualized Absolutist-Constructivist teaching approaches can improve students' attitude towards Mathematics.

Mathematics Learning Experience

Table 4.11
Means and Standard Deviations for Students' Mathematics Learning Experiences Posttest Scores

Teaching Approach Groups	*n*	Mathematics Learning Experiences	
		M	*SD*
Strictly Absolutist	35	4.34	.46
Strictly Constructivist	39	4.42	.45
Contextualized	38	5.00	.46

From the results presented in Table 4.11, it can be concluded that the students in Contextualized Absolutist-Constructivist teaching approach group had better and more engaging learning experiences as revealed by the posttest mean score of *M* = 5.0 (*SD* = .46) compared to the students in the Strictly Absolutist and Strictly Constructivist teaching approach groups with *M* = 4.34 (*SD* = .46) and *M* = 4.42 (*SD* = .45). The students' perception of their learning experiences in the Strict Absolutist and Strict Constructivist groups are nearly similar as revealed by the close mean scores of the students exposed to the teaching approaches.

Table 4.12
One-Way ANOVA of Students' Mathematics Learning Experiences Posttest Mean Scores

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	9.91	4.96	23.82	.000
Within Groups	109	22.67	.208		
Total	111	32.56			

Table 4.12 shows that the posttest mean scores of the students from the three teaching approach groups differ statistically significantly, *F* = (2,109) = 23.82, *p* = *000.* This means that there are at least two groups that are significantly different in terms of students' mathematics learning experiences when they are exposed to the three teaching approaches. To determine which of the three teaching approach groups differ significantly, Tukey's HSD post hoc analyses was performed at .05 level of significance.

To further analyze the extent of difference, the eta squared ($\eta^2 = 0.30$) calculated as the Treatment Sum Squares divided by the Total Sum Squares was obtained. This test statistic indicates a large effect size of the difference among the groups. This implies that 30% of the difference among the posttest mean scores of the students in the three teaching approach groups can be accounted for by the type of teaching approach given to them. The results of the post hoc multiple comparisons are presented in Table 4.13.

Table 4.13
Tukey's HSD Post Hoc Analyses Result of Mathematics Learning Experiences Posttest Mean Scores

(I) Teaching Approach	(J) Teaching Approach	Mean Difference	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	-.08	.106	.741	-.33	.17
	Contextualized	-.67	.107	.000	-.92	-.41
Constructivist	Contextualized	-.59	.104	.000	-.83	-.34

*. The mean difference is significant at the 0.05 level.

Table 4.13 reveals that the posttest mean scores of the students in the Contextualized Absolutist-Constructivist teaching approach group is significantly higher than that of the posttest mean scores of the students exposed to Strict Absolutist (*M* = .67) and Strict Constructivist groups (*M* = .59) at .05 level of significance, *p* = .000, in terms of their learning experiences. This implies that the students in the Contextualized Absolutist-Constructivist teaching approach group had better, more engaging and motivating learning experiences compared to those students in the other two groups.

The Contextualized Absolutist-Constructivist approach to mathematics teaching allows students to be creative and curious in mathematics by encouraging them to device their own way thinking and find various methods to solve mathematics problems. This would foster understanding that mathematics is a product of human involvement and that the influence of human agency in its improvement and perpetuation is undeniable. Lakatos further emphasized that "mathematical activity is human activity… mathematical activity produces mathematics", (Lakatos, 1976 as quoted in Ernest, 1991 p.37). This helps students realize that they are part of mathematics and that they can contribute to its development which in turn engender improved motivation and increased interest in learning mathematics.

The Contextualized Absolutist-Constructivist approach also suggests it is integral in the teaching of mathematics to allow and encourage students to apply what they have learned to practical applications and to the things that interest them. This is supported by the views of Dewey on the importance of bridging the gap between the individual's interest and experiences to encourage effective learning.

Below are some verbatim responses of students in their learning journal emphasizing that the Contextualized Absolutist-Constructivist teaching approach provides worthwhile learning experiences which helps students understand the use and benefits of mathematics:

> Student C: *"The activity is amazing because I realize that the math can be applied in real life."*

Student G: *"The teaching is good. We learned a lot about the lesson. The activities are interesting."*

Student D1: *"I enjoyed this very much. Math is everywhere us sometimes we don't know that."*

However, it was found out that the difference in the mathematics learning experiences posttest mean scores of the students in the Strict Absolutist and Strict Constructivist teaching approach groups (*M* = .08) is not statistically significant, *p* = .741 at .05 level of significance. This implies that the students had similar perception of their learning experiences whether they are exposed to Strict Absolutist or to Strict Constructivist teaching approach. Despite the fact that the teaching approach used in the two groups are greatly the opposite of each other, it was found out that the learning experiences of the students were not very remarkable and satisfying. This may be accounted for in the extreme treatment used in the two teaching approaches, whereas in the Strict Absolutist approach demonstration teaching was used while in the Strict Constructivist approach to mathematics teaching had allowed students to responsibly make sense of what they are learning with very minimal supervision from the teacher.

Thus, it can be concluded from these results that the right mending of the Absolutist and Constructivist perspectives and practices may help students significantly gain better and more positive learning experiences that may increase the students' interest, and improve their achievement in mathematics.

The Contextualized Absolutist-Constructivist perspective in this study posits that mathematics can be regarded as partly found and partly made. It is the mind that processes, analyzes and creates knowledge as the individual make meaning of the learning experience.

Consequently, these mental activities include manipulating and understanding concrete and absolute truths of mathematics concepts. This idea is undergirded by the propositions of Descartes that the mind cannot just create whatever knowledge it so desires. According to him, although it did take mental activities to make knowledge possible, he argues that it was still an uncovering of a preexistent, indubitable type of knowledge.

Mathematics Achievement

The Mathematics Achievement posttest of the students in the three teaching approach groups were a compilation of three (3) long quizzes, where items in the quizzes were matched to items in the developed and validated Mathematics Achievement Test which measured the same concept.

As a result, forty (40) quiz items were matched to the forty (40) items in the Mathematics Achievement Test (see Appendix L: Mathematics Achievement Test and Quiz Items Matching). This was done because the mathematics achievement posttest was not administered due to the sudden community lockdowns implemented to contain the COVID19 virus. However, it is important to note that the quiz items were not validated.

The following sections discuss the analysis procedures done to determine whether the Mathematics Achievement quiz scores of the students in the Contextualized Absolutist-Constructivist teaching approach group is significantly higher than that of the students in the Strict Absolutist and Strict Constructivist groups.

Table 4.14
Means and Standard Deviations of Students' Mathematics Achievement Pretest Scores

Teaching Approach Groups	*n*	Mathematics Achievement	
		M	*SD*
Strictly Absolutist	35	17.66	4.23
Strictly Constructivist	39	19.54	2.69
Contextualized	38	17.58	3.78

Table 4.14 shows that the mean score of 19.54 (*SD* = 2.69) of the students in the Strict Constructivist teaching approach group is the highest pretest mathematics achievement scores among the three groups. This indicates that prior to the teaching experiment done, the students in the Strict Constructivist group had better background knowledge on the topics included in the course Fundamental Concepts of Mathematics Structures.

On the other hand, the students in the Strict Absolutist and Contextualized Absolutist-Constructivist teaching approach groups had close pretest mathematics achievement mean scores with 17.66 and 17.58, (*SD* = 4.23 and 3.78), respectively.

Table 4.15
One-Way Analysis of Variance of Students' Mathematics Achievement Pretest Scores

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	94.02	47.01	3.63	.030
Within Groups	109	1412.84	12.96		
Total	111	1506.86			

Table 4.15 presents that there is a significant difference in the pretest mathematics achievement mean scores of the students across the three teaching approach groups, $F(2,109) = 3.63$, $p = .030$ at .05 level of significance. This means

that the three groups are not comparable prior to the teaching experiment in terms of their background knowledge in the mathematics topics included in this study. Hence, necessary precautions are needed in analyzing the Quiz Achievement mean scores of the students.

Table 4.16
Means and Standard Deviations for Students' Mathematics Quiz Achievement Posttest Mean Scores

		Mathematics Achievement	
Teaching Approach Groups	*n*	*M*	*SD*
Strictly Absolutist	35	26.03	3.85
Strictly Constructivist	39	27.08	3.26
Contextualized	38	30.66	2.98

Table 4.16 shows that the students in the Contextualized Absolutist-Constructivist teaching approach group had the highest mathematics quiz achievement mean score (*M* = 30.66, *SD* = 2.98) than the students in the Strict Absolutist (*M* = 26.03, *SD* = 3.85) and Strict Constructivist (*M* = 27.08, *SD* = 3.26) groups. This indicates that after the students were exposed to the Contextualized Absolutist-Constructivist teaching approach they gained better understanding and learning of the mathematics concepts.

The aim of the Contextualized Absolutist-Constructivist teaching approach of mending the entrenched divide of mathematics philosophy to engender better mathematics teaching was realized. This result is supported by Davison and Mitchelle (2008) who also suggested that the melding of the absolutist and fallibilist perspectives, or philosophical movements may be essential for authentic mathematics learning to occur.

The Contextualized Absolutist-Constructivist perspective to mathematics in this study recognizes that mathematics is a stable body of knowledge that is certain, objective and systematic, but is capable of changing, growing and expanding engendered by continuous utilization and interaction of man to its realm.

The contextualized perspective to mathematics and consequently contextualized approach to mathematics education argues that concepts remain absolute truths and universal until such time its domain validity is expanded to cater to a new knowledge discovered or created. Similar to the ideas of progressive absolutism, contextualized perspective argued that mathematical knowledge proven to be true and absolute is not discarded in the discovery and creation of new and wider mathematical theories. The growth of mathematical knowledge is the result of mathematics utilization aimed to better the way of living.

The results shown in Table 4.16 supports the claim that the Contextualized Absolutist-Constructivist teaching approach has a great potential in aiding students escalate their learning by carefully guiding them learn the skills and concepts and simultaneously engaging them in real-life related mathematics activities and experiences.

Below are some verbatim responses of students in their learning journal proving that the Contextualized Absolutist-Constructivist teaching approach improves students' mathematical thinking and learning. These also supports the claim that effective learning occurs when the concept presented to students is related to their prior knowledge (Ausubel, 1977) and slightly more complicated than their knowledge and capability (Vygotsky, 1986).:

Student B: *"I found the activity engaging. And it makes me think logically."*

Student M: *"The lesson becomes easy to digest as time goes by. The activities are very challenging yet answerable at the same time."*

Student O: *"The activities were awesome because it enhance our brain and knowledge."*

Table 4.17
One-Way Analysis of Covariance of Students' Mathematics Achievement

Source	*df*	*SS*	*MS*	*F*	*p*	Partial Eta Squared
Corrected Model	3	760.30[a]	253.43	29.96	.000	.45
Intercept	1	1504.46	1504.46	177.86	.000	.62
Pretest Exam	1	322.73	322.73	38.15	.000	.26
Groups	**2**	**519.68**	**259.84**	**30.72**	**.000**	**.36**
Error	108	913.56	8.46			
Total	112	89258.00				
Corrected Total	111	1673.86				

a. R Squared = .454 (Adjusted R Squared = .439)

Table 4.17 presents the result of the One-Way ANCOVA conducted to determine whether the mathematics quiz achievement mean scores of the students in the three teaching approach groups differ statistically significantly. It can be observed that the teaching approach used has a significant effect on students' mathematics achievement after controlling students' initial differences in the pretest scores, *F* = (2,111) = 30.72, *p = 000,* at .05 level of significance.

Furthermore, the effect size as reflected by the partial eta squared .36 indicates a large and significant effect based on Cohen's *d.* This further indicates that 36% in the difference in the mathematics quiz achievement mean scores of the students is explained by the teaching approach used.

Table 4.18
Pairwise Comparisons Analyses Result of Students' Mathematics Quiz Achievement Mean Scores

(I) Teaching Approach	(J) Teaching Approach	Mean Difference	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	-.15	.69	1.00	-1.83	1.54
Absolutist	Contextualized	-4.67	.68	.000	-6.32	-3.01
Constructivist	Contextualized	-4.52	.68	.000	-6.17	-2.86

*. The mean difference is significant at the 0.05 level.

Table 4.18 shows that the mathematics quiz achievement mean scores of the students in the Contextualized Absolutist-Constructivist teaching approach group is significantly higher (*M* = 30.66) than that of the mathematics quiz achievement mean scores of the students in the Strict Absolutist (*M* = 26.03) and Strict Constructivist (*M* = 27.08) teaching approach groups, *p* = .000 at .05 level of significance. This supports the claim that the Contextualized Absolutist-Constructivist teaching approach can help students improve their mathematics performance.

However, from the values reflected in Table 4.18, it can be recognized that the mathematics quiz achievement mean scores of the students in the Strict Absolutist and Strict Constructivist teaching approach groups are not significantly different, *p* = 1.00. This informs that the Strict Absolutist and Strict Constructivist teaching approaches are similar in affecting students' learning.

Mathematics conception mean gain scores of students

Table 4.19
t-Test Result Comparing Pretest and Posttest Mathematics Conception Mean Gain Scores

Teaching Approach Groups	Mean Difference		*df*	*t*	*p*	Cohen's *d*
	M	*SD*				
Strict Absolutist	-.02	.19	34	-.501	.620	-.03
Strict Constructivist	-.34	.17	38	-9.45	.000	-.67
Contextualized	-.65	.45	37	-8.92	.000	-1.65

Table 4.19 shows that the difference between the Mathematics Conception pretest and posttest mean scores of students in the Strict Constructivist (*M* = -.34, *SD* = .17) and Contextualized Absolutist-Constructivist (*M* = -.65, *SD* = .45) teaching approach groups are statistically significant, *t*(39) = -9.45, *p* = .000 and *t*(37) = -8.92, *p* = .000, respectively.

The Cohen's *d* effect size of 1.65 standard deviation unit in the mean gain scores of students in the Contextualized Absolutist-Constructivist teaching approach group represents a large effect based on Cohen's rule of thumb and is the highest among the three teaching approach groups. This means that the mean gain scores of the students in the Contextualized Absolutist-Constructivist teaching approach group indicates a large gain and improvement of students' mathematics conception. This further reveals that after exposing the students to the Contextualized Absolutist-Constructivist teaching approach they had better understanding of the nature of mathematics. This means that students realized the benefits of learning the genesis and developmental history of mathematics in improving their appreciation of the importance of the discipline.

Furthermore, the result implies that the Contextualized Absolutist-Constructivist teaching approach to mathematics can help students understand and appreciate more the different facets of mathematics and vast application of its concepts. This result is a significant contribution to theory and the teaching-learning of mathematics since several researches reveal that students have a very narrow conception of the nature and purpose of mathematics, and that these are limited mostly to number concepts and arithmetic (Grootenboer, 2003).

Most students had difficulty talking about the nature of mathematics, and this suggests that many students do mathematics without much thought or opportunity to discuss what it really is (Young-Loveridge, Sharma, Taylor, & Hawera, 2005). The nature of mathematics has significant practical outcomes and implications for the teaching and consequently learning of mathematics (Hersh, 1979; Thompson, 1984).

Hence, it is important to include the explicit teaching of the nature of mathematics in mathematics education. The Contextualized Absolutist-Constructivist teaching approach in mathematics suggested in this study argues that understanding of nature of mathematics can be imparted to students through various and rich ways such as inclusion of discussions on the historical account of mathematical concepts, encouraging students to device own methods and solutions to mathematical problems, and allowing students to apply what they have learned to their interests and passion.

The t-test analysis also reveals that the mean gain scores of the students in the Strict Constructivist teaching approach group has a large standard deviation unit effect size (d = .67) indicating the mean gain obtained by the students in terms of their mathematics conception. However, the students in the Strict Absolutist teaching approach group had no significant mean gain as revealed by the mean difference -

.02 (*SD* = .19), *t*(34) = -.501, *p* = .05. This implies that the Strict Absolutist teaching approach to mathematics does not impart understanding of the nature of mathematics and how they can possibly better learn it with the knowledge of its characteristics and nature. The absolutist view and practice to mathematics teaching cannot improve students' perception and appreciation of mathematics and its vast capabilities.

a. Understanding of the Nature of Mathematics

Table 4.20

Means and Standard Deviations of Students' Mean Gain Scores on Understanding of the Nature of Mathematics

		Understanding of the Nature of Mathematics	
Teaching Approach Groups	*n*	*M*	*SD*
Strictly Absolutist	35	-.01	.20
Strictly Constructivist	39	-.29	.22
Contextualized	38	-.45	.56

Since the values used in the statistical analyses done are the mean gain scores of the students, it indicates a negative value in the direction of the difference. This implies that the greater the negative mean difference is, the higher is the score in the posttest indicative of better gain after exposure to the three teaching approach treatments.

Table 4.20 shows that the students in the Contextualized Absolutist-Constructivist teaching approach group had the highest mean difference of .45 (*SD* = 56) than the Strict Absolutist and Strict Constructivist with .01 and .29 (*SD* = .20, .22), respectively. This proves the claim that the Contextualized Absolutist-Constructivist teaching approach to mathematics can significantly improve and engender better understanding of the nature of mathematics which plays a significant role in fostering students' mathematics appreciation.

Table 4.21
One-Way ANOVA of Students' Mean Gain Scores on Understanding of the Nature of Mathematics

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	3.51	1.75	13.08	.000
Within Groups	109	14.62	.13		
Total	111	18.13			

Table 4.21 shows that the pretest-posttest mean scores difference of the students across groups varies statistically significantly in terms of their understanding of the nature of mathematics, *F* = (2,109) = 13.08, *p = .000.* This implies that at least two of the groups' pretest-posttest mean scores differences are significantly different and that the effect of the teaching approached on students' understanding of the nature of mathematics is largely different and significant. This is further confirmed by the eta squared ($\eta^2 = 0.19$) value which represent the effect size of the treatment. It also means that 19% of the difference on students' pretest-posttest mean scores difference in terms of their understanding of the nature of mathematics is due to the teaching approach used.

Table 4.22
Tukey's HSD Post Hoc Analyses Result of Students' Mean Gain Scores on Understanding of the Nature of Mathematics

(I) Teaching Approach	(J) Teaching Approach	Mean Difference	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	-.28	.085	.004	.08	.48
Absolutist	Contextualized	.43	.086	.000	.23	.64
Constructivist	Contextualized	.16	.083	.154	-.04	.35

*. The mean difference is significant at the 0.05 level.

Table 4.22 presents the results of the multiple comparisons tests done to determine which among the groups varies significantly in terms of students' understanding of the nature of mathematics. Since the data in the analysis includes the mean difference of the pretest and posttest scores, a negative value indicates greater posttest scores than pretest scores.

It can be seen from Table 4.22 that the pretest-posttest mean scores difference of the students in the Contextualized Absolutist-Constructivist teaching approach group is significantly higher than that of the Strict Absolutist (M = .43) teaching group at .05 level of significance, p = .000. This implies that students' understanding of the nature of mathematics improved better when exposed to the Contextualized Absolutist-Constructivist teaching approach. Students had better understanding and appreciation that mathematics changes, improves and grows as opposed to the traditional perception that mathematics is absolute and incorrigible.

The result is relevant to what Perry found out, as discussed by Schommer (2008), that students start off thinking hard facts are handed down by omniscient authority and eventually as they progress in their academics, they start to view knowledge as tentative and coming from reason. This only further undergirds the key point of the Contextualized Absolutist-Constructivist mathematics teaching that it is possible and highly important that students be exposed to the background and developmental stages of mathematical knowledge.

Students also improved perception that solving mathematics can be done in many different ways and that they can be creative and curious when dealing with it. The Contextualized Absolutist-Constructivist teaching approach also helps students appreciate the benefits of learning the developmental history of mathematics concepts in better understanding it. This result supports what Ernest (1991) emphasized about mathematics as a product of human exploration, “As human activity, mathematics cannot be viewed in isolation from its history and its applications in the science and elsewhere.” The Contextualized Absolutist-Constructivist teaching approach also foster understanding that mathematics is a product of human exploration and curiosity rather than an absolute knowledge that awaits discovery.

However, the mean gain scores of students in the Strict Constructivist and Contextualized Absolutist-Constructivist (*M* = .16) teaching approach groups were found to be not statistically significant, *p* = .154. This result indicates that Strict Constructivist and Contextualized Absolutist-Constructivist teaching approaches can improve students’ understanding of the nature of mathematics in almost the same degree.

b. Beliefs about Mathematics Learning and its Use

Table 4.23 presents the descriptive statistics of the mean difference of students’ beliefs about learning mathematics and its use.

Table 4.23
Means and Standard Deviations of Students’ Mean Gain Scores on Beliefs about Mathematics Learning and its Use

Teaching Approach Groups	*n*	Beliefs about Mathematics Learning and its Use	
		M	*SD*
Strictly Absolutist	35	.10	.70
Strictly Constructivist	39	-.23	.26
Contextualized	38	-.70	.51

Table 4.23 shows that the students in the Contextualized Absolutist-Constructivist teaching approach group had higher mean scores difference (*M* = -.70, *SD* = .51) than the Strict Absolutist (*M* = .10, *SD* = .70) and Strict Constructivist groups (*M* = -.23, *SD* = .26) in the direction of the posttest mean score which is indicative of greater score gain on the sub-variable Beliefs in Mathematics Learning and its use.

Furthermore, it can be noticed that the mean score difference of the students in the Strict Absolutist teaching approach group obtained a positive mean difference score. This implies that the pretest mean score is slightly higher than that of the posttest mean score indicative of no mean gain.

Table 4.24
One-Way ANOVA of Students' Mean Gain Scores on Beliefs about Mathematics Learning and its Use

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	11.76	5.88	22.16	.000
Within Groups	109	28.92	.27		
Total	111	40.67			

Table 4.24 shows that the mean scores difference of students across the three teaching approach groups: Strict Absolutist, Strict Constructivist and Contextualized Absolutist-Constructivist varies statistically significantly, $F(2,109) = 22.16$, $p = .000$ at .05 level of significance. This indicates that at least two of the three groups' mean scores difference was significantly different further revealing that the teaching approach used has significant varying effects to students. The computed eta squared effect size ($\eta^2 = 0.29$) indicates a large effect of the teaching approaches used based on Cohen's *d*.

Table 4.25
Tukey's HSD Post Hoc Analyses Result of Students' Mean Gain Scores on Beliefs on Mathematics Learning and its Use

(I) Teaching Approach	(J) Teaching Approach	Mean Difference	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	-.32	.120	.022	.04	.61
Absolutist	Contextualized	.80	.121	.000	.51	1.08
Constructivist	Contextualized	.47	.118	.000	.19	.75

*. The mean difference is significant at the 0.05 level.

The multiple comparison tests result of students' Beliefs on Mathematics Learning and its Use sub-variable as presented in Table 4.25 reveal that the students in the Contextualized Absolutist-Constructivist teaching approach group had statistically significantly higher posttest scores gain (M = -.70) than the students in the Strict Absolutist (M = .10) and Strict Constructivist groups (M = -.23) at the .05 level of significance, *p = .022 and .000,* respectively.

The results imply that students in the Contextualized Absolutist-Constructivist teaching approach group had more positive beliefs about practices and techniques in learning mathematics after they were exposed to the Contextualized Absolutist-Constructivist teaching approach.

Students realize better that mathematics has numerous and important uses and applications as well as simple and personal benefits. They understand that mathematics can help them in their day-to-day reasoning and decision making. They realized that mathematics knowledge can help them better see, understand and appreciate the world they live in. Students also realized that mathematics is something that they can do and can be creative and curious about and not just something that they follow and imitate.

Below are some verbatim responses of students in their learning journal (see Appendix M: Sample Learning Journal Entries) supporting the claim that the Contextualized Absolutist-Constructivist approach to teaching relates to the world they live in:

Student J: *"I think this lesson will help in reasoning for research and when it comes in the field of law. This activity helps me to understand the lesson more."*

Student G: *"The activity is amazing because I realize that math can be applied to real life."*

Student B1: *"There are surprising applications I just learned".*

It can also be recognized from Table 4.25 that the mean scores difference between the Strict Absolutist and Strict Constructivist teaching approach groups in terms of the sub-variable Beliefs in Mathematics Learning and its Use was also statistically significant. This implies that students in the Strict Constructivist teaching approach group had better improvement of their beliefs and practices in learning mathematics than those in the Strict Absolutist group. This further proves that the conventional way of teaching mathematics does not aid students in the improvement of their personal perception of mathematics and how they can possibly learn and understand it better by continually asking them to follow and copy how mathematics is done by the teacher. This amplifies the problematic effects of not recognizing and responding to students' individual learning needs and providing them with rich and meaningful learning opportunities.

Mathematics attitude mean gain scores of students

Table 4.26
t-Test Result Comparing the Mean Gain Scores in Mathematics Attitude

Teaching Approach Groups	Mean Difference		*df*	*t*	*p*	Cohen's *d*
	M	*SD*				
Strict Absolutist	-.18	.14	34	-7.50	.000	-.55
Strict Constructivist	-.35	.17	39	-12.73	.000	-.98
Contextualized	-.60	.24	37	-.15.28	.000	-1.86

The t-test Dependent Paired Sample results presented in Table 4.26 shows that the pretest-posttest mean scores difference of the students in the Contextualized Absolutist-Constructivist teaching approach group was statistically significant $t(37) = -15.28$, $p = .05$ and had the highest effect size as revealed by the Cohen's *d* value ($d = -1.86$). The implication is that there is a large effect size of 1.86 standard deviation unit in the mean scores difference further indicating the large difference between the pretest and posttest mean scores of the students.

The students in the Contextualized Absolutist-Constructivist teaching approach group had a significant improvement in their mathematics attitude after being exposed to the teaching approach. They had more positive perception of mathematics and enthusiasm towards learning and doing mathematics. This proves that the Contextualized Absolutist-Constructivist teaching approach can potentially help students see and appreciate mathematics in more profound ways. This could be accounted for by the more personalized learning practices suggested in the Contextualized Absolutist-Constructivist teaching approach.

Students were able to make more meaningful connections and relations as they engage and start with mathematics concepts that they already know. The

Contextualized Absolutist-Constructivist teaching approach suggests that lessons should be tailored fundamentally recognizing the entry point or background knowledge of the students. Students also had more enthusiasm towards mathematics school works and expressed more appreciation of the use and benefits of mathematics in their lives as reflected in their individual learning journals.

Table 4.26 shows that the pretest-posttest mean scores difference of students in the Strict Absolutist and Strict Constructivist groups also indicate significant gain ($t(34) = -7.50$ and $t(39) = -12.73$, $p = .05$, respectively) and large effect size as revealed by the Cohen's *d* values ($d = -.55, -.98$, respectively). This implies that the traditional lecture-discussion and constructivist teaching approaches can also improve students' attitude towards mathematics. Students appreciate additional knowledge they gained from studying mathematics regardless of the teaching approach used.

a. Affect Dimension

Table 4.27
Means and Standard Deviations of Students' Mean Gain Scores on Attitude Affect Dimension

	Affect Dimension		
Teaching Approach Groups	*n*	*M*	*SD*
Strictly Absolutist	34	-.30	.24
Strictly Constructivist	39	-.77	.37
Contextualized	38	-1.11	.32

The descriptive statistics presented in Table 4.27 shows that the students in the Contextualized Absolutist-Constructivist teaching approach group obtained the highest pretest-posttest mean scores difference ($M = -1.11$, $SD = .32$) compared to the students in the Strict Absolutist (*M* = -.30, *SD* = .24) and Strict Constructivist (*M* = -.77, *SD* = .37) teaching approach groups. This implies that the students in the Contextualized Absolutist-Constructivist teaching approach group had the highest gain in their attitudinal posttest. Students in this group had more positive perception and inclination towards mathematics after being exposed to the Contextualized Absolutist-Constructivist teaching approach. They had more enthusiasm in dealing with and accomplishing mathematics activities. This supports the claim that the Contextualized Absolutist-Constructivist teaching approach has the potential to develop more intrinsic motivation and positive attitude of students towards mathematics by providing them learning opportunities where they can be creative and curious.

In the Contextualized Absolutist-Constructivist teaching approach group students are encouraged to find as many possible methods to solving mathematics problems which in turn aid students develop critical and analytical thinking skills and foster deeper engagement and appreciation of mathematics and its applications.

Table 4.28
One-Way ANOVA of Students' Attitude Mean Gain Scores on Affect Dimension

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	11.46	5.73	56.18	.000
Within Groups	107	10.92	.102		
Total	109	22.38			

Table 4.28 shows that the attitude mean gain scores of the students across the three teaching approach groups are statistically significantly different, $F(2,107) = 56.18$, $p = .000$ at .05 level of significance in terms of the affect dimension. This informs that at least two among the three teaching approach groups' mean gain scores in terms of their attitude towards mathematics are significantly different. This

further implies that there is a significant varying effect caused by the teaching approach used in students' attitude particularly in the affect dimension. The computed eta squared ($\eta^2 = 0.51$) implies that 51% of the change in the posttest mean scores of the students is due to the teaching approach used. Furthermore, this indicates a large effect based on Cohen's *d.*

Table 4.29

Tukey's HSD Post Hoc Analyses Result of Students' Attitude Mean Gain Scores on Affect Dimension

(I) Teaching Approach	(J) Teaching Approach	Mean Difference	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	.46	.074	.000	.29	.64
Absolutist	Contextualized	.80	.076	.000	.62	.98
Constructivist	Contextualized	.34	.073	.000	.16	.51

*. The mean difference is significant at the 0.05 level.

The results of the post hoc tests performed in the mean gain scores on attitude affect dimension of the three teaching approach groups presented in Table 4.29 shows that all the unique pairing of the groups has significantly different mean scores, *p = .000* at .05 level of significance. This implies that the three teaching approaches have significant effects on students' attitude towards mathematics particularly in the affect dimension.

However, it can also be recognized from the results that the mean gain scores of the students in the Contextualized Absolutist-Constructivist teaching approach group (*M* = -1.11) is the highest among the three groups. This indicates that when students are exposed to the Contextualized Absolutist-Constructivist teaching approach they had better improvement in the affect dimension of their attitude towards mathematics. Students became more confident and engaged in mathematics activities when exposed to the Contextualized Absolutist-Constructivist teaching approach. This amplifies the possible potential of the Contextualized Absolutist-Constructivist teaching approach in improving students' perception, attitude and view in mathematics which in turn can help them escalate their mathematics achievement.

b. Behavior Dimension

Table 4.30
Means and Standard Deviations of Students' Mean Gain Scores on Attitude Behavior Dimension

Teaching Approach Groups	*n*	Behavior Dimension	
		M	*SD*
Strictly Absolutist	34	-.15	.18
Strictly Constructivist	39	-.22	.24
Contextualized	38	-.49	.37

Table 4.30 presents the descriptive results on students' Mathematics attitude behavior dimension. It can be observed that in terms of their attitude behavior dimension, students in the Contextualized Absolutist-Constructivist teaching approach group (*M* = -.49, *SD* = .37) obtained the highest mean gain scores compared to the students in the Strict Absolutist (*M* = -.15, *SD* = .18) and Strict Constructivist groups (*M* = -.22, *SD* = .24). The values are negative to indicate the direction of the difference towards the posttest scores of the students. This indicates that the students had improved their observable actions and behaviors towards studying mathematics.

Students in the Contextualized Absolutist-Constructivist teaching approach group allot ample time to study mathematics concepts which in turn led into their better understanding and learning.

Table 4.31
One-Way ANOVA of Students' Attitude Mean Gain Scores on Behavior Dimension

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	2.42	1.21	15.54	.000
Within Groups	107	8.41	.08		
Total	109	10.83			

Table 4.31 shows that the mean gain scores in the behavior dimension of attitude is statistically significantly different, *F*(2,107) = 15.54, *p* = .000 at .05 level of significance. This indicates that there are at least two of the teaching approach groups that has significantly different mean gain scores. The eta squared value ($\eta^2 = .22$) indicating the effect size implies a large effect of the varying teaching approaches used. This means that 22% of the variance in the behavior dimension of attitude can be accounted for by the teaching approach each group was exposed to.

Table 4.32
Tukey's HSD Post Hoc Analyses Results of Students' Attitude Mean Gain Scores on Behavior Dimension

(I) Teaching Approach	(J) Teaching Approach	Mean Difference	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	.07	.065	.555	-.09	.22
Absolutist	Contextualized	.34	.066	.000	.19	.50
Constructivist	Contextualized	.27	.064	.000	.12	.43

*. The mean difference is significant at the 0.05 level.

Table 4.32 shows that the mean gain scores of students in the Contextualized Absolutist-Constructivist teaching approach group (*M* = -.49) is significantly higher than that of the students in the Strict Absolutist (*M* = -.15) and Strict Constructivist groups (*M* = -.22), *p* = .000 at the .05 level of significance. It can be observed that the mean gain scores are negative to denote the direction of the difference where the posttest scores are higher than the pretest scores.

The significant difference in the mean gain scores implies that the students in the Contextualized Absolutist-Constructivist teaching approach group had better improvement in terms of their attitude towards mathematics particularly in their behavior towards the discipline.

Students spent more time in preparing and studying for mathematics tests and keep on solving mathematics problems they find difficult. This indicates a significant improvement in their behavioral attitude towards mathematics which in turn can result to better mathematics learning. Students also learned that they can solve mathematics problems in many different ways which can foster creativity, perseverance and curiosity among students. This supports the claims that the Contextualized Absolutist-Constructivist teaching approach can aid students in realizing that mathematics is not just for the gifted but is doable and reachable by anyone who perseveres on it.

c. Cognitive Dimension

Table 4.33
Means and Standard Deviations of Students' Mean Gain Scores on Attitude Cognitive Dimension

Teaching Approach Groups	*n*	Cognitive Dimension	
		M	*SD*
Strictly Absolutist	34	-.07	.16
Strictly Constructivist	39	-.09	.15
Contextualized	38	-.21	.30

Table 4.33 shows that the students in the Contextualized Absolutist-Constructivist teaching approach group (*M* = -.21, *SD* = .30) obtained the highest mean scores difference than the students in the Strict Absolutist (*M* = -.07, *SD* = .16) and Strict Constructivist (*M* = -.09, *SD* = .15) groups in terms of their attitude particularly in the cognitive dimension. This means that the students in the Contextualized Absolutist-Constructivist teaching approach group had better improvement in their attitude towards mathematics.

Students realized that mathematics can help them improve their way of thinking and reasoning. They further recognized the vast use of mathematics from the simplest concept they apply to their day-to-day lives to the very profound application of mathematics in many other different fields.

Table 4.34
One-Way ANOVA of Students' Attitude Mean Gain Scores on Cognitive Dimension

Source	*df*	*SS*	*MS*	*F*	*p*
Between Groups	2	.42	.21	4.28	.016
Within Groups	108	5.23	.05		
Total	110	5.64			

Table 4.34 shows that there is a statistically significant difference in the attitude cognitive dimension scores of the students in the three teaching approach groups, *F* = (2,108) = 4.28, *p* = .016 at .05 level of significance. This implies that at least two of the three groups' mean gain scores in terms of attitude cognitive dimension differs significantly and that the effects of the teaching approach used varies significantly. Moreover, the eta-squared ($\eta^2 = 0.07$) indicates a large effect of the teaching approach used to students' attitude towards mathematics particularly in

the cognitive dimension. This means that 7% of the significant variation can be accounted for the teaching approach they are exposed to.

Table 4.35
Tukey's HSD Post Hoc Analyses Results of Students' Attitude Mean Gain Scores on Attitude Cognitive Dimension

(I) Teaching Approach	(J) Teaching Approach	Mean Difference	Std. Error	Sig.	95% Confidence Interval	
					Lower Bound	Upper Bound
Absolutist	Constructivist	.01	.052	.975	-.11	.13
Absolutist	Contextualized	.13	.052	.029*	.01	.26
Constructivist	Contextualized	.12	.050	.041*	.00	.24

*. The mean difference is significant at the 0.05 level.

Table 4.35 shows the results of the post hoc tests done which reveals that the attitude on cognitive dimension mean gain scores are statistically significantly different between the Strict Absolutist and Contextualized Absolutist-Constructivist teaching approach groups (p = .029) and between Strict Constructivist and Contextualized Absolutist-Constructivist teaching approach groups (p = .041), at .05 significance level. This implies that the students in the Contextualized Absolutist-Constructivist teaching approach group obtained higher mean gain scores in their attitude particularly in the cognitive dimension than the students in both the Strict Absolutist and Strict Constructivist teaching approach groups. This further indicates that after being exposed to the Contextualized Absolutist-Constructivist teaching approach, the students improved their attitude towards mathematics particularly in the cognitive dimension.

Students realized the importance of mathematics in their day-to-day lives and in appreciating the world they live in. They view mathematics as a significant tool in relating to the things around them and in functioning well in their everyday tasks. This shows that the Contextualized Absolutist-Constructivist teaching approach has the potential to improve students' perception and attitude towards mathematics by exposing them to the undeniable use and applications of mathematics from the simple arithmetic they use every day to its more profound applications.

Correlation between mathematics conception and attitude; understanding of the nature of mathematics and learning appreciation; beliefs about mathematics learning and mathematics learning strategies

a. Mathematics Conception and Attitude

Table 4.36
Spearman's Correlation on Students' Mathematics Conception and Attitude

	Mathematics Conception		
	n	Correlation Coefficient	Sig. (1-tailed)
Mathematics Conception	111		
Attitude	111	.536**	.000

*** Correlation is significant at .01 level (1-tailed)*

Spearman's rank-order correlation was performed to determine the relationship between students' mathematics conception and attitude. Table 4.36 shows that there was a high, positive correlation between students' mathematics conception and attitude, which was statistically significant ($r_s(109) = .536$, $p = .000$) at .01 level of significance. This implies that the better and deeper students understand the nature of mathematics and how they can effectively learn it, the more positive their attitude towards mathematics become. This also indicates that when students understand that mathematics has the ability to change, grow and expand they tend to be more committed and passionate about it.

This result is supported by the results of the study of Schommer (2008), which found out that students who believed mathematical knowledge is linear and unchanging had reported using superficial study habits indicative of low commitment and interest.

Students in the Contextualized Absolutist-Constructivist teaching group realized that mathematics can possibly grow and change over time as new knowledge is created or discovered. This foster a more positive behavior of students towards mathematics because they realized that they can be creative and do mathematics in many different ways. This also informs that teachers have the capability to improve students' attitude towards mathematics which greatly affect their achievement by tailoring to their teaching approaches some practices that impart understanding of the nature of mathematics to students.

b. Understanding of the Nature of Mathematics and Learning Appreciation

Table 4.37

Spearman's Correlation on Students' Understanding of the Nature of Mathematics and Learning Appreciation

	Understanding of the Nature of Mathematics		
	n	Correlation Coefficient	Sig. (1-tailed)
Understanding of the Nature of Mathematics	112		
Learning Appreciation	112	-.133	.081

*** Correlation is significant at .01 level (1-tailed)*

Table 4.37 shows that the result of the Spearman's correlation indicates that there is no significant relationship between students' understanding of the nature of mathematics and their learning appreciation ($r_s(110) = -.133$, $p = .081$) at 0.01 level of significance. This result can be accounted for by students' varying learning experiences in each teaching approach groups. This implies that as students gain better understanding of the nature of mathematics, they also will have better learning appreciation of the discipline.

c. Beliefs about Mathematics Learning and Students' Mathematics Learning Strategies

Table 4.38
Spearman's Correlation on Students' Beliefs about Mathematics Learning and its Use and Mathematics Learning Strategies

	Beliefs about Mathematics Learning and its Use		
	n	Correlation Coefficient	Sig. (1-tailed)
Beliefs about Mathematics Learning and its Use	112		
Mathematics Learning Strategies	112	.031	.373

*** Correlation is significant at .01 level (1-tailed)*

Table 4.38 shows that the result of the Spearman's correlation ($r_s(110) = .031$, $p = .373$) at 0.01 level of significance shows no significant relationship between students' beliefs about mathematics learning and their mathematics learning strategies. This informs that the mathematics learning strategies that students used to deal with mathematics has no relationship with their beliefs on how to learn it.

In the dissertation of Anthony (1994) it was found out that there is a strong indication that the appropriateness and effectiveness of mathematics learning strategies of students relate to the learning goal and task demands. This implies that students' mathematics learning strategies vary widely as the learning goals and mathematics activities vary. Students adjust their mathematics learning strategies and cope momentarily on the given learning objective and task.

Mathematics conception, attitude and learning experiences positive predictors of mathematics achievement

The assumptions required to perform Multiple Regression are all satisfactorily met by the data set (see Appendix N: Multiple Regression Assumptions Checking Results). The following sections discuss the results of the analyses.

Table 4.39
Linear Regression Model for Mathematics Achievement Dependent Variable

Model	*R*	*R* Squared	Adjusted *R* Squared	Std. Error of the Estimate
1	.36[a]	.13	.10	3.66

a. Predictors: (Constant), Mathematics Conception, Attitude, Learning Experiences
b. Dependent Variable: Mathematics Achievement

As Table 4.39 shows, the *R* multiple correlation coefficient of .36 indicates a moderate good level of prediction. The R^2 value of .13 indicates that 13% of the variability in the dependent variable (mathematics achievement) is due to the independent variables (mathematics conception, attitude, learning experiences).

This low R^2 value can be accounted for by the fact that the regression model measures the effects of students' understanding of the nature of mathematics, attitude and learning experiences to their mathematics achievement which are variables that measures human perception and behavior. It is expected for fields that measure human behaviors such as psychology, and other social sciences to have R^2 lower than 50%.

Furthermore, since the Residual Plot of the regression follows a normal distribution hence it is a good model to predict students' mathematics achievement.

Table 4.40
ANOVA Results of the Multiple Linear Regression

Model	Sum of Squares	df	Mean Square	*F*	Sig.
Regression	206.57	3	68.86	5.15	.002[b]
Residual	1430.53	107	13.37		
Total	1637.10	110			

a. Dependent Variable: Mathematics Achievement
b. Predictors: (Constant), Mathematics Conception, Attitude, Learning Experiences

Table 4.40 presents the ANOVA table of the regression. The *F*-ratio informs whether the overall regression model is a good fit for the data set in this study. It can be observed from Table 4.40 that students' mathematics conception, attitude and learning experiences (independent variables) statistically significantly predict students' mathematics achievement (dependent variable), $F(3, 107) = 5.15$, $p = .0005$.

Table 4.41
Unstandardized Coefficients of the Multiple Regression Analysis Model

Model	Unstandardized Coefficients		Standardized Coefficient	*t*	Sig.	95.0% Confidence Interval for *B*	
	B	Std. Error	Beta			Lower Bound	Upper Bound
(Constant)	14.92	5.73		2.60	.011	3.56	26.27
Conception	.099	.033	.316	2.99	.003	.033	.165
Attitude	.019	.037	.054	.513	.609	-.055	.093
Learning Experiences	-.025	.025	-.090	-.987	.326	-.075	.025

The unstandardized coefficients presented at Table 4.41 indicate how much the mathematics achievement varies with an independent variable when all other independent variables are held constant. It can be observed from Table 4.41 that for every 1 score increase in the mathematics conception there is a .099 increase in the mathematics achievement of students.

Similarly for every 1 score increase in the attitude score of students there is a .019 increase in the mathematics achievement of students. However, the unstandardized coefficient for learning experiences indicates that for every 1 score increase in the learning experiences of students, there is a -.025 decrease in mathematics achievement.

It can be further observed from Table 4.41 that the significant relationship indicator and the multiple regression *beta* coefficient have opposite signs. This indicates that learning experiences variable is a suppressor variable in the regression model which further implies that although the learning experiences variable is positively correlated with the dependent variable-mathematics achievement, higher score in the learning experiences will result to lower mathematics achievement.

To further analyze and understand this result, it is helpful to reflect on the part of the learning experiences variable that is independent of the other two predictors, namely mathematics conception and attitude, and the part on which it is negatively related to the dependent variable mathematics achievement.

It can be realized that the learning experiences variable directly look into the students' perception of their learning experiences in the three teaching approach groups which were largely different from each other caused by the teaching approaches used.

In general, the form of the equation to predict mathematics achievement is given by: (.099 x conception) + (.019 x attitude) – (.025 x learning experiences)

Furthermore, it can be observed from the table that only the mathematics conception among the independent variables tested statistically significantly different from zero. This indicates that changes in mathematics conception will equally implies similar change in the mathematics achievement.

In summary, a Multiple Linear Regression analysis was performed to predict mathematics achievement based on students' mathematics conception, attitude towards mathematics and their learning experiences. The resultant regression equation was found to be statistically significant with $F(3, 107) = 5.15$, $p = .0005$ with an R^2 of .13. Moreover, students' predicted mathematics achievement is equal to 14.92 + (.099 x conception) + (.019 x attitude) – (.025 x learning experiences) where mathematics achievement is measured as quiz scores and the three independent variables – mathematics conception, attitude and learning experiences were measured using the sum of responses in a Likert scale.

Students' mathematics achievement will increase .099 for every 1 unit increase in the mathematics conception and .019 increase for every 1 unit increase in attitude, however there is a .025 unit decrease in mathematics achievement for every .025 unit increase in learning experiences. It was further found out that mathematics conception is statistically significantly a predictor of mathematics achievement. This implies that improving students' mathematics conception could mean an improvement in the students' mathematics achievement.

However, attitude and learning experiences found do not contribute significantly to the prediction of mathematics achievement. This could be accounted for by the fact that the students participating in the study are in their third year BS Mathematics program.

Several studies showed that as the individual progresses in school there is a significant possibility that they will have less positive attitude towards mathematics (Mazana, M.Y., et al, 2019, Kibrislioglu, 2015; Mazana, Montero, & Casmir, 2019; Arslan, Yavuz, & Karatas, 2014). The participants in the study are students in the tertiary level and under the BS in Mathematics program. The results reveal that as students become more expose to mathematics, they tend to have lesser positive attitude towards mathematics despite performing well in mathematics.

Reconceptualized Framework

Based on the findings of this study, the following Reconceptualized Framework shows the significant effects and relationships among the variables under study. Figure 4.1 below presents the Reconceptualized Framework as a result of this study.

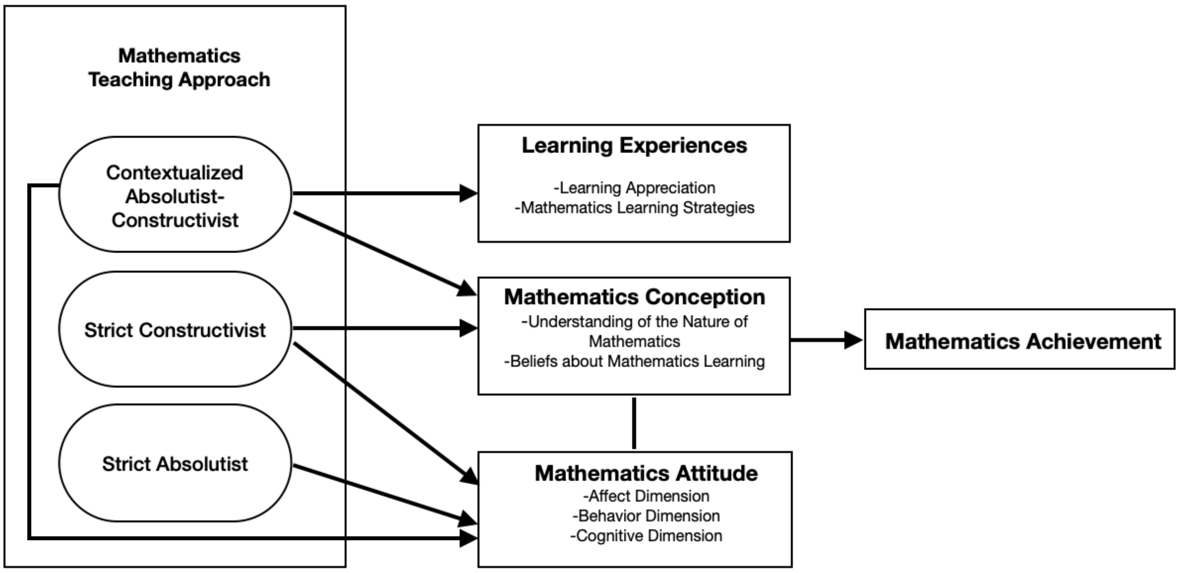

Figure 4.1. Reconceptualized Framework of the Study

It can be seen from Figure 4.1 that the Contextualized Absolutist-Constructivist teaching approach has significant effects to students' mathematics learning experiences, conception and attitude. This result supports the claim that the Contextualized Absolutist-Constructivist teaching approach can foster students' understanding of the nature of mathematics and their beliefs of how to learn and use it.

Furthermore, the Contextualized Absolutist-Constructivist teaching approach improved students' attitude towards mathematics which engender increased engagement and learning responsibility. The results also undergird the claim that the Contextualized Absolutist-Constructivist teaching approach enrich students' learning experiences by providing them with mathematics learning opportunities that encourage them to be curious and creative.

The Strict Constructivist teaching approach also improved students' mathematics conception and attitude. This can be accounted for by the way the students were allowed to discover and explore mathematics with minimal supervision. However, the results showed that the students in the Strict Constructivist teaching approach group had no significant change in their perception of their mathematics learning experiences even after being exposed to the Strict Constructivist teaching approach.

The result can be accounted for by the confusion and frustration students experience when they are left in their own devices to figure out how to cope or make sense of the mathematics given to them despite recognizing their background knowledge and presenting the mathematics concepts in scaffolded way.

On the other hand, the results revealed that the Strict Absolutist teaching approach also significantly affect students' attitude towards mathematics. This may be accounted for by the guidance the students receive as they make meaningful relation of mathematics. However, the Strict Absolutist learning approach does not significantly affect students' mathematics conception and learning experiences.

The results of this study also reveal that students' mathematics conception and attitude have significant, high positive relationship. This may be due to the fact that as students improve their understanding of the nature of mathematics and how to effectively learn mathematics their attitude and behavior towards the discipline also improved. The findings of this study further revealed that mathematics conception is a significant predictor of students' mathematics achievement.

Chapter V

SUMMARY, CONCLUSION AND RECOMMENDATION

This chapter presents the summary of findings, conclusions and recommendations drawn from the results of this study.

Summary of Findings

1. The pretest comparison of the students in terms of their mathematics conception revealed that the three intact learning groups – Strict Absolutist, Strict Constructivist and Contextualized Absolutist-Constructivist – are comparable.

   The mathematics conception posttest mean scores of the students in the Contextualized Absolutist-Constructivist teaching approach group was the highest among the teaching approach groups which indicates that students exposed to the Contextualized Absolutist-Constructivist approach had better learning of mathematics concepts.

   Furthermore, it was found from the multiple comparisons of the mean scores that the mathematics posttest mean scores of the students in the Contextualized Absolutist-Constructivist teaching approach group is significantly higher than those of the Strict Absolutist and Strict Constructivist groups. The computed effect size revealed that 21% of the difference in the mathematics conception posttest mean scores of the students can be accounted for by the teaching approach used in each group. The posttest mean scores of the students in the Strict Absolutist and Strict Constructivist teaching approach groups were found not statistically significantly different. This informs that students obtained similar mathematics posttest mean scores indicating that the two teaching approaches may have similar effect in students' mathematics conception.

In terms of students' mathematics attitude pretest mean scores, the three teaching approach groups were also found to be statistically comparable. On the other hand, examining the posttest mean scores of the students in the three teaching approach groups revealed that students in the Contextualized Absolutist-Constructivist teaching approach group had statistically significantly higher mathematics attitude posttest mean scores. And the effect size of 25% was indicative of a large effect of the teaching approaches used in the mathematics attitude mean scores difference of the students.

This implies that students in the Contextualized Absolutist-Constructivist teaching approach group had more positive attitude towards mathematics after they were exposed to the Contextualized Absolutist-Constructivist teaching approach. This further indicates that students' attitude towards mathematics improved significantly when they were explicitly taught of the nature of mathematics, history of the concepts and the practical and sophisticated applications of mathematics. This supports the claim that the Contextualized Absolutist-Constructivist approach to mathematics teaching has the potential to improve students' attitude towards mathematics. However, it was found that the mathematics attitude posttest mean scores of the students in the Strict Absolutist and Strict Constructivist teaching approach groups has no significant difference.

In terms of students' mathematics learning experiences it was found that students' in the Contextualized Absolutist-Constructivist teaching approach group had higher posttest mean scores than the other groups which indicates that students exposed to the Contextualized Absolutist-Constructivist teaching approach had more engaging and motivating classroom activities and opportunities. The eta square value revealed a large effect of 30% indicating the percentage effect of the teaching approach used. The mathematics learning experiences posttest mean scores of the students in the Strict Absolutist and Strict Constructivist teaching approach groups were not statistically significantly different. This reveals that students will have similar perception of their learning experience whether they are exposed to Strict Absolutist or Strict Constructivist teaching approach.

Lastly, the mathematics achievement pretest scores of the students across the three teaching approach groups were found to be statistically significantly different with students in the Strict Constructivist teaching approach group having the highest. This means that before the start of the experiment in this study, the students in the Strict Constructivist teaching approach group had deeper understanding of the mathematics concepts and better background knowledge in mathematics.

Hence, necessary precautions were taken in the analysis of the mathematics achievement posttest scores of the students by using the pretest scores as covariate. It was found that the mathematics achievement mean scores of the students in the Contextualized Absolutist-Constructivist teaching approach group is significantly higher than those of the students in the other two groups. This reveals that the Contextualized Absolutist-Constructivist teaching approach has the potential to help students escalate and improve their mathematics learning. The mathematics achievement mean scores of the students in the Strict Absolutist and Strict Constructivist teaching approach groups are found to be not significantly different. Thus, indicating that students' mathematics achievement will be the same whether they are exposed to Strict Absolutist or Strict Constructivist teaching approach.

2. The mean gain scores of the students in the Strict Constructivist and Contextualized Absolutist-Constructivist teaching approach groups were tested statistically significantly different in terms of their understanding of the nature of mathematics with that of the Contextualized Absolutist-Constructivist teaching approach group being the highest.

Moreover, the Cohen's *d* effect size of the mean gain scores of students in the Contextualized Absolutist-Constructivist teaching approach group represent a large effect based on Cohen's rule of thumb and is the highest among the three teaching approach groups. This implies that students' understanding of the nature of mathematics in the Contextualized Absolutist-Constructivist Reconceptualized teaching approach group improved remarkably after they were exposed to the Absolutist-Constructivist Reconceptualized approach of teaching mathematics.

Furthermore, students evidently had better understanding of the nature of mathematics, genesis and developmental stages of important mathematical concepts and the significant applications of mathematical ideas and knowledge.

However, students in the Strict Absolutist teaching approach group had no significant mean gain that only confirms that the traditional lecture-discussion method of teaching mathematics does not impart understanding of the nature of mathematics to students. The students continue to perceived mathematics as a set of rules that they only need to follow which may deter students' further appreciation of mathematics.

The statistical analysis done to delve deeply into students' understanding of the nature of mathematics revealed results that confirms the claim that the Contextualized Absolutist-Constructivist teaching approach explicitly and effectively informs students of the nature of mathematics which is important in students' engagement with mathematics. However, it was found that the Strict Constructivist and Strict Absolutist teaching approaches has no statistically significant effect when it comes to teaching students of the nature of mathematics.

In terms of the variable beliefs in mathematics learning and its use, it was found that students in the Contextualized Absolutist-Constructivist teaching approach group had the highest statistically significant mean gain than the two other teaching approach groups. This implies that after exposing students to the Contextualized Absolutist-Constructivist teaching approach students significantly improve their mathematics learning beliefs and had remarkable realization of the use and benefits of mathematics. The computed effect size also revealed a large effect indicating that the type of teaching approach use can greatly influence students' beliefs of mathematics learning and its use.

3. The three teaching approach groups had significantly improved students' mathematics attitude with the students in the Contextualized Absolutist-Constructivist teaching approach group having the highest mean gain. In terms of the affect dimension of attitude, it was found that the three teaching approach groups have significant varying effects on students' mathematics attitude. The computed effect size indicates that a large significant difference in students attitude affect dimension can be attributed to the teaching approach used.

Moreover, the students in the Contextualized Absolutist-Constructivist teaching approach group obtained the highest mean gain indicating greater improvement in terms of the affect dimension. Students developed more positive attitude towards mathematics after being exposed to the Contextualized Absolutist-Constructivist approach to teaching mathematics. Students became more confident and passionate in dealing with mathematics and completing mathematics tasks. This amplifies the possible potential of the Contextualized Absolutist-Constructivist teaching approach as an alternative approach in teaching mathematics which guide students into becoming more adept, passionate and incline to mathematics.

Assessing students' attitude behavior dimension, the results revealed that all the three teaching approaches significantly improved students' attitude behavior dimension. The computed effect size also indicates a large effect of the teaching approach used in students' attitude behavior dimension difference. The students in the Contextualized Absolutist-Constructivist teaching approach group obtained the highest significant mean difference among the groups which implies that the Contextualized Absolutist-Constructivist approach to teaching mathematics can significantly improve students' behavior towards mathematics.

Students allot more time studying mathematics, especially in those concepts they find difficult to grasp at first. They tend to persevere more after realizing that mathematics can be solved in different ways, which in turn engender curiosity and creativity among students. This undergirds the claim that the Contextualized Absolutist-Constructivist teaching approach can encourage students to think critically and creatively emphasizing that they can do mathematics without solely adhering to the methods presented to them. Evidently students became more engaged in mathematics activities and showed more enthusiasm in completing mathematics tasks after being exposed to the Contextualized Absolutist-Constructivist teaching approach.

In terms of the cognitive dimension of attitude, it was found that students in the Contextualized Absolutist-Constructivist teaching approach group had significantly higher mean gain than those of the students in the Strict Absolutist and Strict Constructivist teaching approach groups.

However, there was no significant difference between students' mean gain in the Strict Absolutist and Strict Constructivist teaching approach groups. This indicates that the Contextualized Absolutist-Constructivist teaching approach can significantly improve students' attitude cognitive dimension. After being exposed to the Contextualized Absolutist-Constructivist teaching approach students realized the undeniable practical use and remarkable applications of mathematics. This engendered students to engage more in mathematics tasks. Students realized that mathematics is not just for the gifted which in turn encourage more engagement and perseverance among students.

4. The results revealed that there was a strong positive correlation between students' mathematics conception and attitude towards mathematics. This implies that the better and deeper students understand the nature of mathematics and how they can effectively learn it the more positive their attitude towards mathematics become. Students realized that there are new concepts being discovered or created in mathematics that results to its expansion and growth. They further learned that there are interesting concepts in mathematics that are still in its developmental stage. This encourages students to think more critically

which in turn can encourage contribution to mathematics development. This informs teachers that students can be creative with mathematics and encourage them to recognize students as mathematics explorers and possible mathematics contributors rather than passive recipient of mathematics that only follows rules.

It was also revealed from the results that there is no significant relationship between students' understanding of the nature of mathematics and their learning appreciation. This explains that regardless of how well and inform students are about the nature of mathematics and the different effective ways of learning it, students' learning appreciation will not be affected. Students' varying learning experiences may have caused this result. This informs that apart from imparting information on the nature of mathematics, teachers should tailor mathematics activities that provide students more positive, motivating and engaging learning experiences.

Lastly, it was found that students' beliefs in learning mathematics and their learning strategies have no significant relationship. This implies that students' mathematics learning strategies is not influence by their beliefs in how they can learn it. This implies that students' mathematics learning strategies vary widely as the learning goals and mathematics activities vary. Students adjust their mathematics learning strategies and cope momentarily on the given present learning objective and task.

5. It was found that mathematics conception is a strong positive predictor of mathematics achievement. This implies that the better students understand the nature of mathematics and how they can possibly effectively learn it the better they perform in mathematics. This supports the claim that understanding the nature of mathematics can significantly aid students escalate their learning.

Hence, it is encouraged to provide students learning opportunities in mathematics where they can explicitly and implicitly learn and understand the nature of mathematics.

However, it was revealed by the results of this study that students' attitude and learning experiences are not predictors of mathematics achievement. This implies that students' attitude and learning experiences will not influence how they perform in mathematics. This can be accounted for by the amount of exposure to mathematics students already have. Several researches revealed that as individuals progress to education, the less positive their attitude towards mathematics become. The participants in the study are college students majoring in mathematics. Moreover, it is important to note that the mathematics achievement posttest items used in this study were items from the quizzes which were not validated. Each quiz item was instead matched with the item that measure the same concept or skill in the mathematics achievement pretest whose items were validated. This was done in lieu of the unadministered mathematics achievement posttest caused by the sudden class suspensions and community lockdowns due to the COVID-19 pandemic.

Conclusions

Based on the findings of the study, the following conclusions are drawn:

1. The Contextualized Absolutist-Constructivist alternative approach to mathematics teaching significantly improve students' mathematics conception. Students realize and understand the nature of mathematics and how they can effectively learn it. Furthermore, the Contextualized Absolutist-Constructivist

approach to mathematics teaching improved students' mathematics achievement better than the Strict Absolutist and Strict Constructivist teaching approaches.

2. The Strict Constructivist and Contextualized Absolutist-Constructivist teaching approaches significantly improve students' understanding of the nature of mathematics and beliefs in mathematics learning. However, the Strict Absolutist teaching approach cannot enhance students' understanding of the nature of mathematics.

3. The Strict Absolutist, Strict Constructivist and Contextualized Absolutist-Constructivist teaching approaches significantly improve students' attitude towards mathematics. The Contextualized Absolutist-Constructivist teaching approach has the highest significant contribution in the change in students' attitude towards mathematics.

4. There was a high and positive relationship between students' mathematics conception and attitude. The more students understand the nature of mathematics the more positive their attitude towards mathematics become. However, students' understanding of the nature of mathematics and their learning appreciation has no relationship.

5. Students' mathematics conception positively influence students' mathematics achievement. As students better understand and appreciate the nature of mathematics, the better they perform in mathematics. However, students' attitude and learning experiences do not affect students' mathematics achievement.

Recommendations

Based on the conclusions, the recommendations of the study are:

1. The Contextualized Absolutist-Constructivist alternative approach to mathematics teaching which aims to provide students mathematics learning opportunities that teach and expose them to the nature of mathematics and how to effectively learn it can be considered in mathematics classrooms. The result of this study can encourage Mathematics educators to rethink their traditional way of Mathematics teaching and embrace more individualized and student-centered methods. Specific teaching methodologies such as seamless tailoring of historical accounts of mathematics concepts, encouraging students to be creative in mathematics, and allowing students to apply what they have learned to the things that interest them foster better mathematics learning. Hence, it is recommended that these mathematics teaching methodologies can be used more often in mathematics classrooms.

2. Further studies on the nature of mathematics and its relevance to mathematics education can be done to reveal other information that can help and improve mathematics teaching and learning. Studies on how to effectively incorporate nature of mathematics in teaching and assessment can be further done. The Strict Absolutist or the traditional lecture-discussion method of teaching mathematics can be used less often in mathematics classrooms. The teaching of mathematics can be made engaging by providing students with learning activities that encourage them to explore and be creative in mathematics.

3. The Contextualized Absolutist-Constructivist teaching approach can be used in mathematics classrooms to improve students' attitude towards mathematics.

4. Mathematics activities and learning opportunities can be tailored by integrating the teaching of the nature of mathematics and how to effectively learn it to consequently improve students' attitude towards mathematics. This can be

done by incorporating the teaching of the historical account of mathematics concepts into lesson teaching, encouraging students to be creative in their mathematics solutions, design mathematics tasks in real-world scenarios, and allow students to apply what they have learned in their areas of interests and passion.

5. A similar study using pretest-posttest analysis which will use the same validated achievement test can be done to confirm the results of this study. Future research results can slightly vary since the posttest instrument used in this study is different from that of the pretest. It is further recommended that similar studies be done with longer period of implementation, such as one full semester for tertiary level and involving non-mathematics major students to validate the results of this study.

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Appendices

APPENDIX A

CLASS SCHEDULE (Part 1)

BSM CS 1B

BULACAN STATE UNIVERSITY

Class Section: BSM CS 1B - G1
Academic Year/Term: 2019-2020 2nd Semester Campus: Main Campus

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
06:00 AM							
06:10 AM							
06:20 AM							
06:30 AM							
06:40 AM							
06:50 AM							
07:00 AM		MAT 105 FH CS-AR / Reyes, Jo Ann V. - sched 1 -	MCS 103 AS / Law Bldg. Marcos, Marcus Louis - sched 1 -	MAT 104 FH 207 / Malang, Paulino - sched 1 -	MAT 105 FH 207 / Reyes, Jo Ann V. - sched 2 -	MAT 106 FH 104 / Mangaran, Armela - sched 2 -	NSTP 11 Ronquillo, Rose Ann T. - sched 1 -
07:10 AM							
07:20 AM							
07:30 AM							
07:40 AM							
07:50 AM							
08:00 AM							
08:10 AM							
08:20 AM							
08:30 AM						MAT 104 FH 207 / Malang, Paulino - sched 2 -	
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09:00 AM							
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09:20 AM							
09:30 AM							
09:40 AM							
09:50 AM							
10:00 AM		FID 101 FH AVR-A / Chua, Geraldine S. - sched 1 -	MCS 104 AS / Salamo, Ma. Paulita - sched 1 -		FID 101 FH AVR-A / Chua, Geraldine S. - sched 1 -		
10:10 AM							
10:20 AM							
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10:40 AM							
10:50 AM							
11:00 AM							PE 11 Panganiban, Jessie - sched 1 -
11:10 AM							
11:20 AM							
11:30 AM		UTS 101 FH CS-AR / Canare, Kaileen R. - sched 1 -		MAT 106 FH 104 / Mangaran, Armela - sched 1 -	UTS 101 FH AVR-A / Canare, Kaileen R. - sched 2 -		
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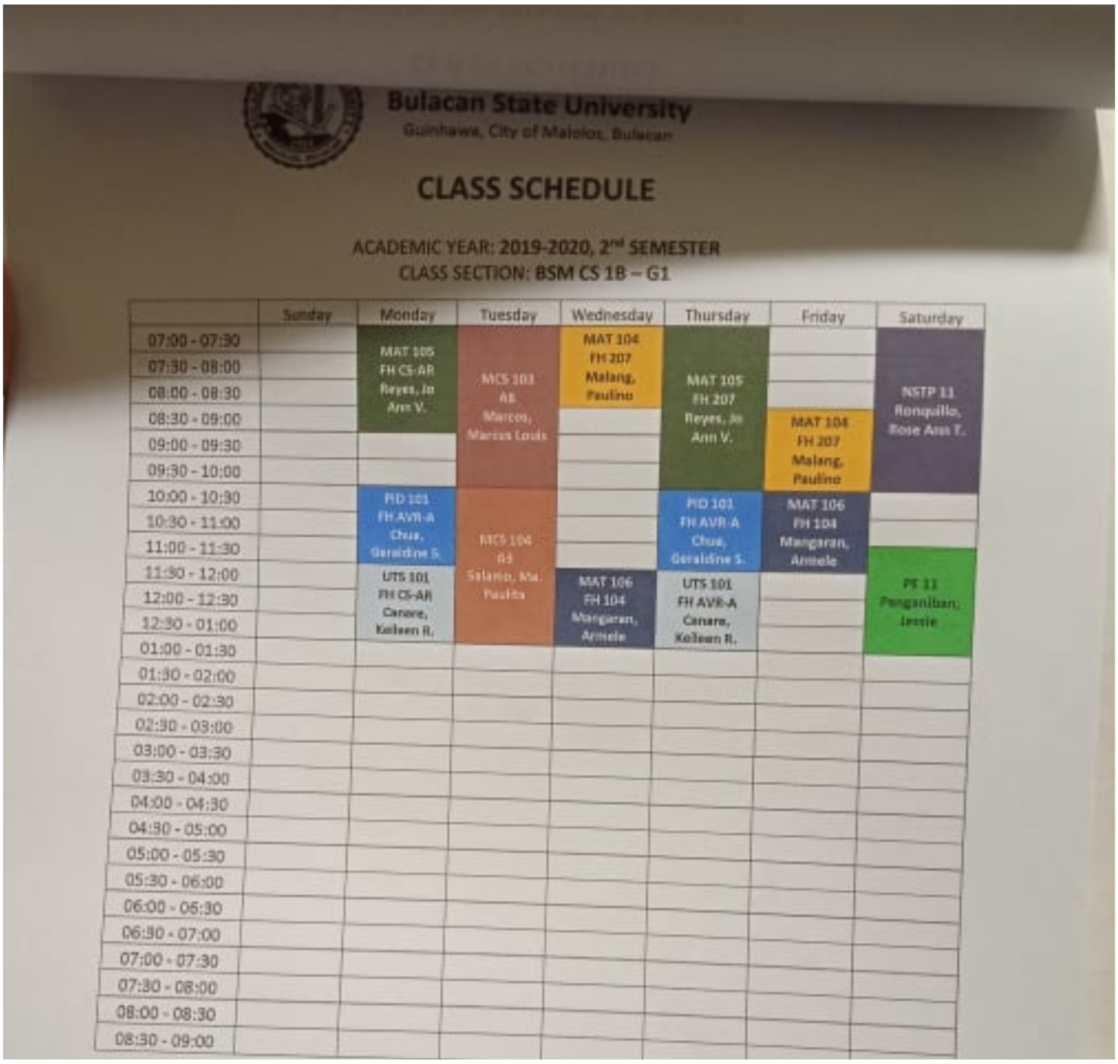
SIGNED CLASS SCHEDULE

BSM CS 1B



Bulacan State University
Guinhawa, City of Malolos, Bulacan

CLASS SCHEDULE

ACADEMIC YEAR: 2019-2020, 2nd SEMESTER
CLASS SECTION: BSM CS 1B – G1

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
07:00 - 07:30		MAT 105 FH CS-AR Reyes, Jo Ann V.	MCS 103 A8 Marcos, Marcus Louis	MAT 104 FH 207 Malang, Paulino	MAT 105 FH 207 Reyes, Jo Ann V.		NSTP 11 Ronquillo, Rose Ann T.
07:30 - 08:00							
08:00 - 08:30							
08:30 - 09:00						MAT 104 FH 207 Malang, Paulino	
09:00 - 09:30							
09:30 - 10:00							
10:00 - 10:30		PID 101 FH AVR-A Chua, Geraldine S.	MCS 104 A9 Salamo, Ma. Paulita		PID 101 FH AVR-A Chua, Geraldine S.	MAT 106 FH 104 Mangaran, Armele	
10:30 - 11:00							
11:00 - 11:30							PE 11 Panganiban, Jessie
11:30 - 12:00		UTS 101 FH CS-AR Canare, Kelleen R.		MAT 106 FH 104 Mangaran, Armele	UTS 101 FH AVR-A Canare, Kelleen R.		
12:00 - 12:30							
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07:00 - 07:30							
07:30 - 08:00							
08:00 - 08:30							
08:30 - 09:00							

BSM CS 1D

BULACAN STATE UNIVERSITY

Class Section: BSM CS 1D - G1
Academic Year/Term: 2019-2020 2nd Semester Campus: Main Campus

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
06:00 AM 06:10 AM 06:20 AM							
06:30 AM 06:40 AM 06:50 AM							
07:00 AM 07:10 AM 07:20 AM			PE 11 Tayao, Antonino - sched 1 -	MAT 105 FH CS-AR / Reyes, Jo Ann V. - sched 2 -	MCS 103 A6 / Law Bldg. Marcos, Marcus Louis - sched 1 -		
07:30 AM 07:40 AM 07:50 AM							
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08:30 AM 08:40 AM 08:50 AM						MAT 106 FH 104 / Manganan, Armale - sched 1 -	
09:00 AM 09:10 AM 09:20 AM							
09:30 AM 09:40 AM 09:50 AM		MAT 105 FH CS-AR / Reyes, Jo Ann V. - sched 1 -					
10:00 AM 10:10 AM 10:20 AM			UTS 101 FH AVR-A / Rotaquio, Mariome - sched 1 -	MAT 104 FH 207 / Malang, Paulino - sched 1 -	MCS 104 A3 / Salamo, Ma. Paulita - sched 1 -	MAT 104 FH 207 / Malang, Paulino - sched 1 -	NSTP 11 NH 510 / Castro, Ariel - sched 1 -
10:30 AM 10:40 AM 10:50 AM							
11:00 AM 11:10 AM 11:20 AM							
11:30 AM 11:40 AM 11:50 AM		PID 101 FH AVR B / Chua, Geraldine S. - sched 1 -	MAT 106 FH 104 / Manganan, Armale - sched 2 -	PID 101 FH AVR B / Chua, Geraldine S. - sched 2 -		UTS 101 FH CS-AR / Rotaquio, Mariome - sched 2 -	
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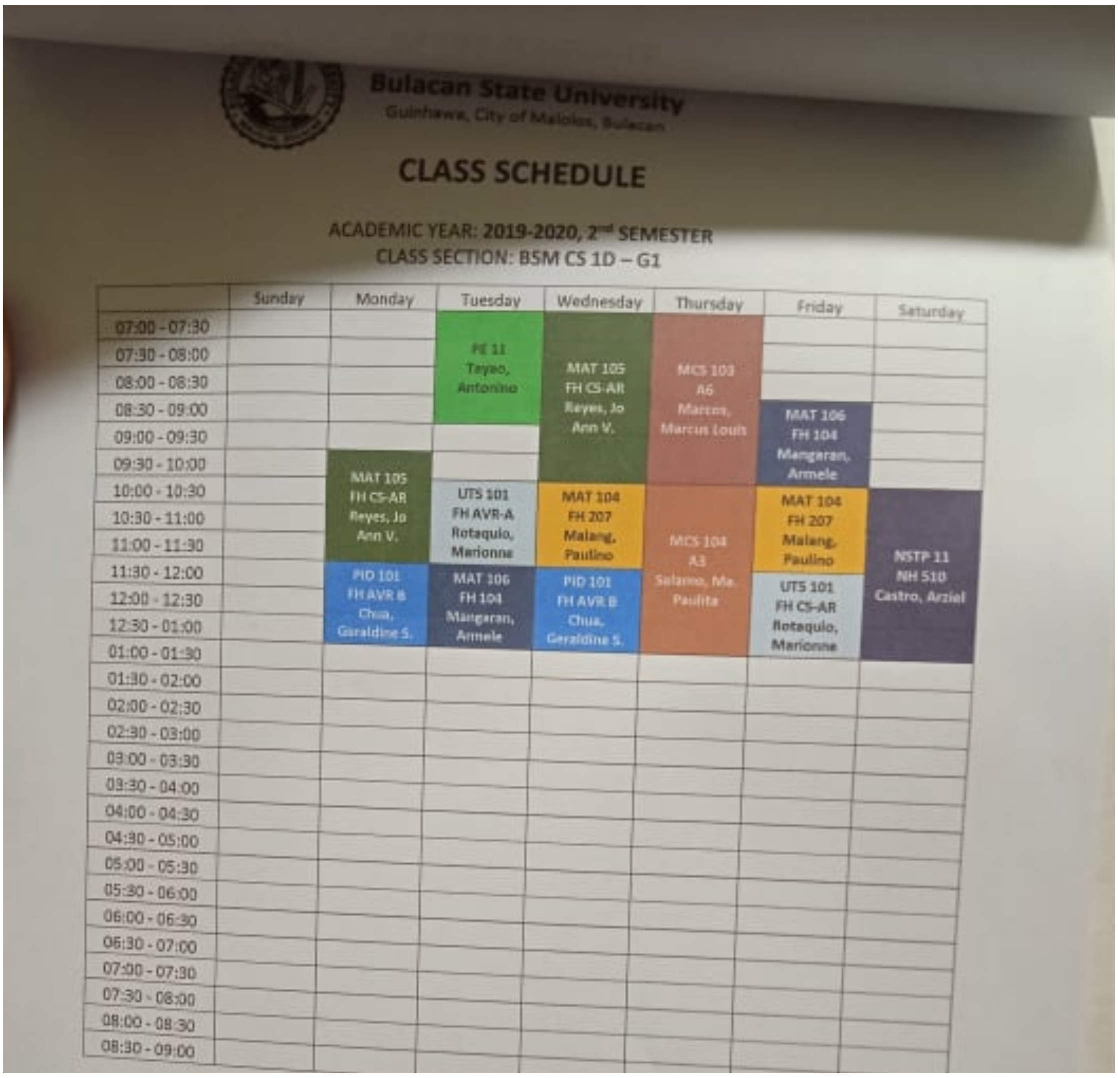
BSM CS 1D



CLASS SCHEDULE

ACADEMIC YEAR: **2019-2020, 2nd** SEMESTER
CLASS SECTION: BSM CS 1D – G1

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
07:00 - 07:30			PE 11 Tayao, Antonino	MAT 105 FH CS-AR Reyes, Jo Ann V.	MCS 103 A6 Marcos, Marcus Louis		
07:30 - 08:00							
08:00 - 08:30							
08:30 - 09:00						MAT 106 FH 104 Mangaran, Armele	
09:00 - 09:30							
09:30 - 10:00		MAT 105 FH CS-AR Reyes, Jo Ann V.					
10:00 - 10:30			UTS 101 FH AVR-A Rotaquio, Marionne	MAT 104 FH 207 Malang, Paulino	MCS 104 A3 Solano, Ma. Paulita	MAT 104 FH 207 Malang, Paulino	NSTP 11 NH 510 Castro, Arziel
10:30 - 11:00							
11:00 - 11:30							
11:30 - 12:00		PID 101 FH AVR B Chua, Geraldine S.	MAT 106 FH 104 Mangaran, Armele	PID 101 FH AVR B Chua, Geraldine S.		UTS 101 FH CS-AR Rotaquio, Marionne	
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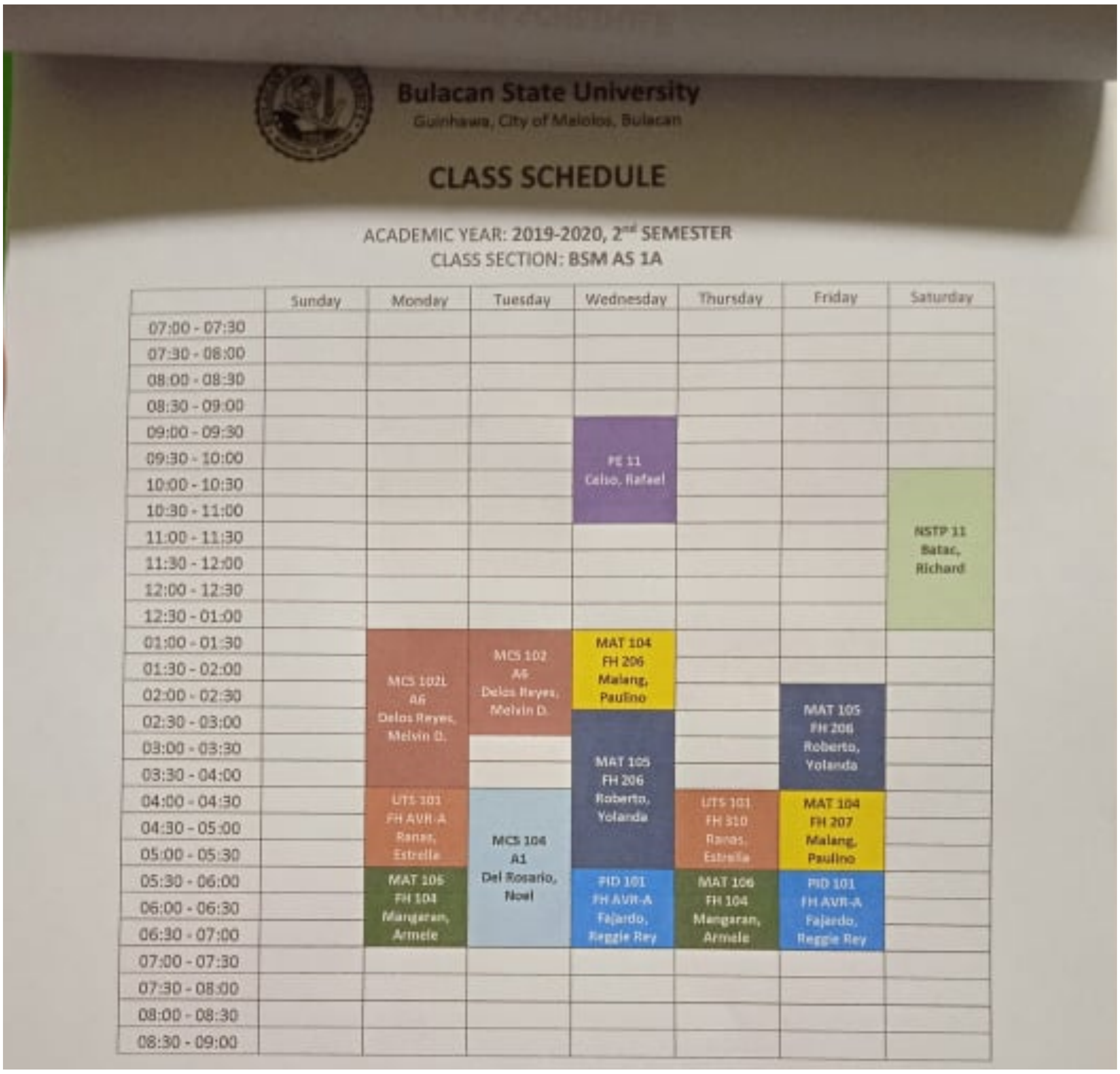
BSM AS 1A



Bulacan State University
Guinhawa, City of Malolos, Bulacan

CLASS SCHEDULE

ACADEMIC YEAR: 2019-2020, 2nd SEMESTER
CLASS SECTION: BSM AS 1A

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
07:00 - 07:30							
07:30 - 08:00							
08:00 - 08:30							
08:30 - 09:00							
09:00 - 09:30				PE 11 Celso, Rafael			
09:30 - 10:00							
10:00 - 10:30							NSTP 11 Batac, Richard
10:30 - 11:00							
11:00 - 11:30							
11:30 - 12:00							
12:00 - 12:30							
12:30 - 01:00							
01:00 - 01:30		MCS 102L A6 Delos Reyes, Melvin D.	MCS 102 A6 Delos Reyes, Melvin D.	MAT 104 FH 206 Malang, Paulino			
01:30 - 02:00							
02:00 - 02:30						MAT 105 FH 206 Roberto, Yolanda	
02:30 - 03:00				MAT 105 FH 206 Roberto, Yolanda			
03:00 - 03:30							
03:30 - 04:00							
04:00 - 04:30		UTS 101 FH AVR-A Ranas, Estrella	MCS 104 A1 Del Rosario, Noel		UTS 101 FH 310 Ranas, Estrella	MAT 104 FH 207 Malang, Paulino	
04:30 - 05:00							
05:00 - 05:30							
05:30 - 06:00		MAT 106 FH 104 Mangaran, Armele		PID 101 FH AVR-A Fajardo, Reggie Rey	MAT 106 FH 104 Mangaran, Armele	PID 101 FH AVR-A Fajardo, Reggie Rey	
06:00 - 06:30							
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BSM AS 1A

BULACAN STATE UNIVERSITY

Class Section: BSM AS 1A
Academic Year/Term: 2019-2020 2nd Semester Campus: Main Campus

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
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10:00 AM							NSTP 11 Batac, Richard - sched.1 -
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01:00 PM		MCS 102L A2 / Dela Reyes, Melvin D. - sched.1 -	MCS 102 A6 / Law Bldg. Dela Reyes, Melvin D. - sched.1 -	MAT 104 FH 206 / Malang, Paulino - sched.1 -			
01:30 PM							
02:00 PM						MAT 105 FH 206 / Roberto, Yolanda - sched.2 -	
02:30 PM				MAT 105 FH 206 / Roberto, Yolanda			

BULACAN STATE UNIVERSITY

Class Section: BSM AS 1A
Academic Year/Term: 2019-2020 2nd Semester Campus: Main Campus

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
03:00 PM		MCS 102L A2 / Dela Reyes, Melvin D. - sched.1 -		MAT 105 FH 206 / Roberto, Yolanda - sched.1 -		MAT 105 FH 206 / Roberto, Yolanda - sched.2 -	
03:30 PM							
04:00 PM		UTS 101 FH AVR-A / Ramos, Estrella - sched.1 -	MCS 104 A1 / Del Rosario, Noel - sched.1 -		UTS 101 FH 310 / Ramos, Estrella - sched.2 -	MAT 104 FH 207 / Malang, Paulino - sched.2 -	
04:30 PM							
05:00 PM							
05:30 PM		MAT 106 FH 104 / Mangaran, Armela - sched.1 -		PID 101 FH AVR-A / Fajardo, Reggie Ray - sched.1 -	MAT 106 FH 104 / Mangaran, Armela - sched.1 -	PID 101 FH AVR-A / Fajardo, Reggie Ray - sched.1 -	
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APPENDIX B
FCM COURSE SYLLABUS

Republic of the Philippines
Bulacan State University
City of Malolos, Bulacan
Tel/Fax (044) 791-0153

Office of the Dean of Instruction

COURSE SYLLABUS
2nd Semester, AY 2019-2020

COLLEGE : SCIENCE **DEPARTMENT : MATHEMATICS**
COURSE CODE : MATH 213B **COURSE TITLE :FUNDAMENTAL CONCEPTS OF MATHEMATICS**
FACULTY : RAINILYN LEONARDO-DUQUE
CONSULTATION HOURS : By appointment

COURSE DESCRIPTION:

The course deals with the elementary theory of sets, set notations, set operations and the use of Venn diagram, relations that involved equivalence relations and equivalence classes and partial ordering, digraph of relation and matrices. In particular, the course deals with the true-false statement, compound statements involving logical connectives, the study of valid and invalid arguments, logical implications and equivalences, sequences and series and mathematical induction. Techniques in proving include the use of truth values and truth tables, methods of proof, and methods of showing the validity of an argument, counting techniques, and their applications to computer science and information technology. Integers, binary operations and introduction to mathematical systems are also included.

LEARNING OUTCOMES:

On completion of the course, the student is expected to be able to do the following:

ATTRIBUTES OF IDEAL GRADUATE (AIG)	LEARNING OUTCOMES
Highly Competent	LO1: Have a solid foundation of the mathematical theories and concepts in computer science, information's technology, statistics and business applications LO2: Demonstrate understanding on basic concepts of mathematics and their applications LO3: Apply the concepts of sets to open and closed statements in proving valid arguments and in proving identities LO4: Identify solutions for effective working environment.

Ethical Professional	LO5: Understand the mathematical concepts and its properties
Service-Oriented	LO6: Use results to give recommendations on existing problems.
Contribute to country's sustainable growth and development	LO7: Relate social issues and cultural standards in collecting data and evaluating results necessary as professional. LO8: Demonstrate the practical application of fundamental concepts of mathematics

FINAL COURSE OUTPUT:

As evidence of attaining the above learning outcomes, the student is required to do and submit the following during the indicated dates of the term.

RUBRIC FOR ASSESSMENT:

Criteria	Exemplary 4	Satisfactory 3	Developing 2	Beginning 1
Strategy/ Procedure	Uses an efficient and effective strategy to solve the problem	Uses an effective strategy to solve the problem	Uses a strategy to solve the problem but it is not effective	Does not use a strategy to solve the problem
Discussion/ Explanation	Discussion/ explanation is detailed and clear.	Discussion/ explanation is detailed and clear.	Discussion/ explanation is a little difficult to understand but includes critical components.	Discussion/ explanation is difficult to understand.
Mathematical Concepts	Shows complete understanding of the mathematical concepts used to solve the problem	Shows substantial understanding of the mathematical concepts used to solve the problem	Shows some understanding of the mathematical concepts needed to solve the problem.	Shows very limited understanding of the mathematical concepts needed to solve the problem
Mathematical Terminology and Symbols	Advanced, correct terminology and symbols are used, making it very easy to understand what was done.	Correct terminology and symbols are used, making it very easy to understand what was done.	Correct terminology and symbols are used, but it is sometimes not easy to understand what was done	There is little use, or a lot of inappropriate use, of terminology and symbols.
Neatness and Organization	The work is presented in a neat, detailed and organized fashion that is	The work is presented in a neat and organized fashion that is	The work is presented in an organized fashion but maybe hard to	The work appears sloppy and unorganized. It is hard to

	easy to read.	easy to read	read	know what information goes together.

OTHER REQUIREMENTS AND ASSESSMENTS:

Aside from term examinations, the student will be assessed at other times during the term by the following: seat works, group exercises, assignments and long tests.

GRADING SYSTEM:

$$LT_1 + LT_2 + \ldots . + LT_n + ME = TRANSMUTED = MG$$

$$LT_1 + LT_2 + \ldots . + LT_n + FE = TRANSMUTED = TFG$$

$$FG = \frac{MG + TFG}{2}$$

NOTES:

- Other activities such as exercises, seat works, problem sets, assignments, etc. maybe converted into Long Test as a preference of the instructor
- Points for Attendance and recitation are bonus points (add-on)

GRADES	PERCENTAGE	DESCRIPTIVE RATING
1.0	97-100	Excellent
1.25	94-96	Excellent
1.5	91-93	Very Good
1.75	88-90	Very Good
2.0	85-87	Good
-2.25	82-84	Good
2.5	79-81	Satisfactory
2.75	76-78	Satisfactory
3.0	75	Passed
4.0	Lacking Requirements	Conditional
5.0	74 and Below	Failed

LEARNING EPISODES:

Learning Outcomes	Topics	WEEK	Learning Activities
Comprehend the vision, mission and goals of the University. Demonstrate support to and uphold the vision, mission,	Orientation of the vision, mission goals, and objectives of the university and institution	1	Lecture/Class Discussion

goals and objectives in maintaining high quality education in relation t0o one's performance in the classroom.			
LO1	Propositions Logical Connectives and Truth Tables Negation Conjunction Disjunction Conditional and Bi-conditional Proposition Tautologies and Contradiction Logical Equivalence	2	Group Activity / Working with Exercises
LO2	Argument Method of Deduction Propositional Identities Formal Proof of Validity Rule of Conditional Proof Rule of Indirect Proof	3	Group Activity / Working with Exercises
LO3-LO4	Set Notation Two Methods of Writing Sets and Kinds of Sets	4-5	Seatwork/ Recitation/ Board work
LO2	Venn-Euler Diagram, Operations on Sets, Applications of the Union and Intersection of Sets and Computer Representation of Set	5-6	Group Activity /Board work
LO5	Laws of Algebra of Sets, Principle of Duality of Sets and Proving Set Identities and	6-7	Group Activity / Working with Exercises
LO6	Integers Divisibility Division Algorithm	7	Seatwork/ Board work
LO5	Euclidean Algorithm Proving Mathematical Induction	8	Seatwork/ Research/ Board work
	Midterm Examination	9	

LO6	Domain and Range of Relations Methods of Describing Relations Identity and Inverse Relation Composition of Relations Digraph of the Relation Path of Length *n* Properties of Relation Closure Properties Partitions Equivalence Relation Equivalence Relation and Partition Cross Partitions *n*-ary Relation Function Notation One-to-one and Onto Functions Evaluation of Functions Operations of Functions Inverse Functions Composition of the Functions *f* and *g* Odd/Even Functions	10-12	Seatwork/ Board work
LO7	Binary Operations Modular Operations Properties of Matrices Matrix Operations Inverse Matrix Application of the Inverse Matrix Operation on Complex Numbers	13-14	Group Reporting/ Recitation
LO8	Mathematical System Semi-Group Group Ring Fields	15-17	Group Activity / Working with Exercises
	Final Term Examination	18	

TEACHING METHODOLOGIES:

To utilize technology in the subject a courseware will be used in facilitating the subject. Each topic will be compose of the following : Introduction, Objectives, Discussion, Summary, Evaluation and Enrichment Activities.

The classroom discussion will also play a vital role as a teaching method. Class participation will be highly encouraged to enhanced students’ communication skills and awareness of the topic. Additional point will be given to the students who exerted extra effort for class discussion.

Students are expected to read the required textbook prior to class. The class activity will include interaction between the professor and the students. Class activities will involve group interactions to clarify concepts and reinforce learning. Some assignments and students helps will be made available online.

REFERENCES:

Textbook/ Workbook

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Lipschutz, Seymour and Marc Lipson (2007). *Discrete mathematics, 3rd ed*. USA: McGraw-Hill Inc.

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Ymas Jr., Sergio E., Orlando E. Esparrago, Priscilla Belen Coligado, and Dolores G. Bacelon. (2007). *Discrete mathematics*. Manila: Sta Monica Printing Corp.

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Ocampo, Shirley R. (2007). *Counting techniques: counting without enumerating*. A Paper presented to the seminar at the Holy Angels University, Angeles City, Pampanga.

Pascasio, Arlene A. (2007). *The mathematics in domino toppling*. A paper presented to the seminar at the Holy Angels University, Angeles City, Pampanga.

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He, M. , et. al. (2009). Discrete structures web course material. Retrieved February 21, 2011 from http://www.cs.odu.edu/~toida/nerzi/content/web_course.html.

Ensley D., et. al. Discrete math resources. Retrieved May 5, 2012 from http://webspace.ship.edu/deensley/DiscreteMath/flash/.

Discrete math videos in college math help. Retrieved April 13, 2010 from http://www.mathvids.com/topic/mathhelp/20-discrete-math.

CLASSROOM POLICIES:

1. Prompt and regular attendance to the class is required
2. A student is considered tardy if he/she arrives in the class 15 minutes after it starts.
   Meanwhile, if he/she arrived in the class 20 minutes after the start of the scheduled time,

He/she will be considered absent, However if he/she wishes to enter and stay in the class,
He/she will be allowed, except that he/she will be marked absent.

3. Absences may be excused for any of the following reasons:
    a. Official representation in curricular, co-curricular activities approved by the university
    b. Sickness duly certified by the attending doctor or our university physician
    For the absence(s) incurred due to any aforementioned reasons, student is held responsible to comply with all assignments and for contents of the coursed missed.
4. Absences incurred more than 9 hours, excluding excused absences, can be automatically dropped according to the guidelines of the university rules. Meanwhile, student will not be allowed to drop once the midterm examination was taken.
5. Internal Classroom and Examination Rules
    a. The use of mobile phone and other electronic gadgets are strictly prohibited during class discussion and examination. Violation of this rule will be penalized of the confiscation of these gadgets and written explanation to the guidance counselor or any college authority (Area Chair, Program Chair, Assistant Dean or Dean of the College)
    b. Only calculators will be allowed as a computing tool during examination
    c. Borrowing of calculator while the examination is going on is strictly prohibited
    d. Anyone caught cheating during the examination will automatically get a grade of 5.0
    e. Examination missed due to absences will be covered by a special examination upon presentation of excused letter. Special examination can be taken within seven (7) days from the day that the examination was held
    f. Avoid eating inside the classroom as much as possible , however, if cannot be avoided, please be responsible enough to bring out the residues or waste and throw it properly to the garbage
    g. Refrain from talking to your classmates /seatmates while the discussion or examination is ongoing. Communication with your classmates/seatmates during the examination will be considered cheating.
    h. Any clarifications/queries during the examination must be directly communicated with your proctor/instructo

PREPARED BY:

____
RAINILYN L. DUQUE

RECOMMENDING APPROVAL:

____
ENGR. MA. CONCEPCION D.C. ARELLANO
Program Chair, Mathematics

APPROVED:

____
RICHARD F. CLEMENTE, Ph.D
Dean, College of Science

Declaration

I have read and understood the above syllabus in full and in participating in this course I agree to the above rules. I have a clear understanding of the policies and my responsibilities, and I have discussed everything unclear to me with the instructor.

I will adhere to the academic integrity policy and treat my fellow students with the due respect.

I understand that I have to provide proper documentation as soon as possible to be eligible to make-ups for missed exams, and that I can only make up a lab during the same day.

I understand that this syllabus can be modified or overruled by announcements of the instructor in class or on any social media site at any time.

________________________ ____________________ ______________

Print your name Signature Date

Student's copy

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Declaration

I have read and understood the above syllabus in full and in participating in this course I agree to the above rules. I have a clear understanding of the policies and my responsibilities, and I have discussed everything unclear to me with the instructor.

I will adhere to the academic integrity policy and treat my fellow students with the due respect.

I understand that I have to provide proper documentation as soon as possible to be eligible to make-ups for missed exams, and that I can only make up a lab during the same day.

I understand that this syllabus can be modified or overruled by announcements of the instructor in class or on any social media site at any time.

________________________ ____________________ ______________

Print your name Signature Date

Instructor's copy

Please detach the instructor's copy of the declaration and submit to your instructor during the first week.

APPENDIX C

Sample Lessons

The table below presents a comprehensive example of lessons implementing the Strict Absolutist, Strict Constructivist and the Absolutist-Constructivist Reconceptualized teaching approach.

Summary of a Lesson Structure Implementing Strict Absolutist, Strict Constructivist and the Reconceptualized Learning Approaches

Topic 1: Logic
Objective: At the end of the lesson, the students should be able to:
a. test the validity of an argument;
b. translate symbolic representation to verbal form and vice-versa; and
c. appreciate the art of testing the validity of arguments.

AC Reconceptualized Approach	Strictly Absolutist (Conventional) Approach	Strictly Constructivist Approach
Stage 1. The teacher will deliver the discussion on the different rules of inferences and give examples of each. The examples will be tailored to students' interests and answers to previous Reflection Exit Cards (REC).	Recall through recitation.	Students will be asked to form small groups to discuss and work on a "warm-up" set of problems which will review their existing knowledge in arguments and Rules of Inferences.
The teacher will conduct direct instruction allowing students to gain understanding of the concepts of Rules of Inferences.	Lecture-discussion method.	

Stage 2. The teacher will use flexible grouping allowing students to group themselves as they find it effective. Students will be asked to construct their own set of arguments and let other groups to validate whether their arguments are valid or not using formal proof of validity. The groups that will exchange their set of works will be combine together to discuss each other's work.	Students will be given a set of arguments and they will validate the truthfulness of each using formal proofs.	Students will be shown with a video where they will take down the arguments being presented and solve whether the arguments are valid or not using formal proofs. They will work on this in small groups. Link: https://www.youtube.com/watch?v=HyMO0lUHkck https://www.youtube.com/watch?v=UTm75PK8uSs (Students will work on this together with minimal guidance from the teacher. However, they are allowed to ask questions.)
Stage 3. Students will be asked to create a real-life scenario which will involve presentation of arguments, for example a court proceeding, a debate, etc. Students will be asked to explore an example in the area that interest them. They will then be instructed to construct their arguments and the formal proof of validity for each argument. Students will present their works to the class to showcase their learning and creativity.	The students will show their works on the board.	Students will be asked to create a video tutorial which aims to share what they have learn in formal proofs of validity to other students.

Example:

<u>The Crime Operatives in Action</u>
(students who are very much interested in suspense and crime thriller solving stories)

Present a proof to determine whether the following is a valid argument or not.

"If the police do not catch the murderer within a week, then there will be a public outcry. If there is public outcry, then the chief of the police will resign. The chief of police will not resign. Therefore, the police will catch the murderer within a week.

Topic 2: Geometric Transformation (Reflection)
Objective: At the end of the lesson, the students should be able to:

a. understand the concept of reflection;
b. solve problems involving reflection transformation; and
c. recognize reflection patterns.

AC Reconceptualized Approach	Strictly Absolutist (Conventional) Approach	Strictly Constructivist Approach
Stage 1. The teacher will deliver the discussion on the topic transformation reflection tailoring students' prior knowledge on perpendicular bisector and symmetry. (This will be based on the Reflection Exit Card (REC) answers of the students from their previous lesson.)	Recall through recitation.	Students will be grouped.
The teacher will conduct direct instruction allowing students to gain understanding of the concepts of transformation reflection.	Lecture-discussion method.	Students will be given worksheets with several reflection pattern examples to study and observe.
Stage 2. The teacher will use flexible grouping allowing students to group themselves as they find it effective. Students will be given a worksheet they will work on together. This worksheet includes real-life problems involving reflection concepts. Questions will be structured in such a way	Students will be given drill exercises to practice what they have learned as the teacher demonstrated it on the board.	They will be asked to formulate/construct the ideas and concepts of transformation reflection based on the examples in the worksheet. (Students will work on this together with minimal guidance from the teacher. However, they are allowed to ask questions.)

that students will see the relevance of what they are learning to that of the things outside their classroom.		
Stage 3. Students will be asked to create a design using reflection and work on this individually. Students will choose what to work on based on their interest and passion. Students will be encouraged to use the software Geogebra to help them create artistic designs. Example: *"Choose your own adventure"*	Board activities/recitation can also be done allowing students to show their solutions.	Students will then be given exercise/set questions to solve. This will include asking students to recognize reflection patterns they see around them.
Interested in architecture – Create tile designs that uses reflection pattern. **Interested in interior designs** – Create a wall paper design using reflection pattern. **Interested in sports/fashion** – Create jersey or clothing design with reflection pattern detail. **Interested in tattoo** (culture) – Create artistic tattoo designs using reflection pattern.	Assignment will be given at the end of the class period.	Students will be asked to share to class what they have learned.

APPENDIX D

CLASSROOM OBSERVATION FORM

Section: ____________________ Date: __________

Time: ______________________ Room: _________

Teaching Method: **Absolutist-Constructivist Reconceptualized Perspective to Teaching**

Part I. Please rate the following items in each classroom aspect using the scale below.

Rating Scale: 5 – Very Evident
4 – Moderately Evident
3 –
2 – Slightly Evident
1 – Not Evident

Class Aspects	Tasks	1	2	3	4	5
Discussion/ Lesson Presentat	Clear discussion relating previous knowledge to the topic to be discussed					/
	Delivers concepts in a simple and clear way					/
	There is smooth transition or flow of the discussion.					/
Lesson Objective	Lesson objectives are achieved in the discussion.					/
	Lesson aims are also achieved through students' activities.					/
Learner participation	Students are engaged during discussion through smart probing and questioning.					/
	Students are participative and collaborating with each other.					/
	Students' interests are tapped through the activities.					/
Classroom Management	Students are well-seated and discipline.					/
	The teacher makes eye contact with students.					/
	The teacher monitor students' activities.					/

Part II. Please narrate or describe how partially absolutist and constructivist method of mathematics teaching was delivered differently from the other two teaching methods under study in terms of the following: lesson discussion methodology, additional ideas and concept incorporated to the lesson, students' mathematics activities.

Students was asked to create a design using reflection and work on this individually. Students were taking on a particular role, develop a product for a specified audience in a particular format and on a topic that gets right at the heart of what matters most in a particular segment of study.

IRISH T. BALDEVARONA
Chair, Mathematics Department

CLASSROOM OBSERVATION FORM

Section: ___________________ Date: __________
Time: ______________________ Room: _________
Teaching Method: **Strict Constructivist Teaching**

Part I. Please rate the following items in each classroom aspect using the scale below.

Rating Scale: 5 – Very Evident
4 – Moderately Evident
3 –
2 – Slightly Evident
1 – Not Evident

Class Aspects	Tasks	1	2	3	4	5
Discussion/ Lesson Presentat	Clear discussion relating previous knowledge to the topic to be discussed					/
	Delivers concepts in a simple and clear way				/	
	There is smooth transition or flow of the discussion.					/
Lesson Objective	Lesson objectives are achieved in the discussion.					/
	Lesson aims are also achieved through students' activities.					/
Learner participation	Students are engaged during discussion through smart probing and questioning.					/
	Students are participative and collaborating with each other.					/
	Students' interests are tapped through the activities.					/
Classroom Management	Students are well-seated and discipline.					/
	The teacher makes eye contact with students.					/
	The teacher monitor students' activities.					/

Part II. Please narrate or describe how constructivist method was delivered differently from the other two teaching methods under study in terms of the following: lesson discussion methodology, additional ideas and concept incorporated to the lesson, students' mathematics activities.

This method let students to understand, solve and recognized the geometric transformations. Students was given worksheets with several reflection pattern examples to study and observe. They also asked to formulate or construct the ideas and concepts of transformation reflection based on the examples given in the worksheet.

IRISH T. BALDEVARONA
Chair, Mathematics Department

CLASSROOM OBSERVATION FORM

Section: ____________________ Date: __________

Time: ______________________ Room: _________

Teaching Method: **Traditional Method**

Part I. Please rate the following items in each classroom aspect using the scale below.

Rating Scale: 5 – Very Evident
4 – Moderately Evident
3 –
2 – Slightly Evident
1 – Not Evident

Class Aspects	Tasks	1	2	3	4	5
Discussion/ Lesson Presentat	Clear discussion relating previous knowledge to the topic to be discussed				/	
	Delivers concepts in a simple and clear way					/
	There is smooth transition or flow of the discussion.					/
Lesson Objective	Lesson objectives are achieved in the discussion.					/
	Lesson aims are also achieved through students' activities.				/	
Learner participation	Students are engaged during discussion through smart probing and questioning.					/
	Students are participative and collaborating with each other.					/
	Students' interests are tapped through the activities.					/
Classroom Management	Students are well-seated and discipline.					/
	The teacher makes eye contact with students.					/
	The teacher monitor students' activities.					/

Part II. Please narrate or describe how traditional method was delivered differently from the other two teaching methods under study in terms of the following: lesson discussion methodology, additional ideas and concept incorporated to the lesson, students' mathematics activities.

Teachers convey knowledge to their students primarily through discussing the lessons and demonstrate how geometric transformation is done while students listen and watch the teacher. Students then are given drill exercises to work on.

IRISH T. BALDEVARONA
Chair, Mathematics Department

APPENDIX E

Permission Letter

20 November 2019

Dr. Edgardo M. Santos
Vice President for Academic Affairs
Bulacan State University

Dear Sir:

Greetings!

I, Rainilyn Leonardo-Duque, a regular faculty of the College of Science-Mathematics Department and a bona fide doctorate student of the University of the Philippines Open University, is currently embarking on dissertation about a philosophical reconceptualization of mathematics and mathematics education, titled "Absolutist-Constructivist Mathematics Teaching: Effects on Students' Mathematics Conception, Attitude, Performance and Learning Experiences.

In line with this, I would like to seek your permission to allow me empirically practice the educational implications of the said alternative perspective through teaching in collaboration with the instructor who would be assigned to teach the course Fundamental Concepts of Mathematical Structure during the Second Semester of the Academic Year 2019-2020. The length of the team teaching will be from start of the semester until the midterm examination period which will be approximately 27 hours and will involve three (3) freshmen classes of BS Math students in the College of Science. This classroom implementation will be of great importance to the success of the research study being delved on and to the betterment of mathematics education, in general.

Rest assured that research and teaching ethics will be observed and adhered to during the conduct of the classroom teaching and data gathering procedures. Also, students' performance and participation will be well documented, recorded and reported to the instructor assigned in the course.

I am prayerfully hoping for your positive response on this matter. Thank you very much for your consideration and support.

Respectfully yours,

Rainilyn Leonardo-Duque

Noted by:

IRISH T. BALDEVARONA
Chair, Mathematics Department

Endorsed by:

VIVIEN M. TALISAYON, Ph.D.
Dissertation Adviser, UP Open University

RICHARD F. CLEMENTE, Ph.D.
Dean, College of Science

Approved by:

EDGARDO M. SANTOS, Ph.D.
Vice President, Academic Affairs

APPENDIX F

Student Mathematics Conception Inventory (SMCQ)

First Draft (For Expert Validation)

IMPORTANCE	RELEVANCE	CLARITY
3 – Essential	4 – Very relevant	4 – Very clear
2 – Useful but not essential	3 – Relevant but need minor revision	3 – Clear but need minor revision
1 – Not Necessary	2 – Item need some revision	2 – Item need some revision
	1 – Not relevant	1 – Not clear

Nature of Mathematics Understanding		IMP	REL	CLA	Remarks
1	Mathematics is primarily a formal way of representing the real world.				
2	In mathematics, something is either right or wrong.				
3	Mathematics problems can be done correctly in only one way.				
4	Mathematics concepts exist to help man improve his life.				
5	Existing mathematical knowledge can be changed, revised or expanded.				
6	Mathematics never changes.				
7	Mathematics has practical and functional use and purpose.				
8	Mathematics is a tool that other disciplines such as science use to advance their concepts.				
9	Mathematics is a product of human exploration and curiosity.				
10	Mathematics is a set of abstract concepts such as formulas that bear no relationship with the world.				
11	Mathematics is a set of correct and formal rules.				
12	Abstract and complex mathematical concepts are later on used for practical applications.				
13	Mathematical knowledge is capable of changing, growing and expanding as new concepts are discovered or created.				
14	Mathematics is a science of pattern finding and generalizations.				
15	New math concepts may be discovered or created that contrast previously held truths.				
16	History of mathematical knowledge bears no importance in understanding it today.				
17	There are several other ways or methods in solving some mathematics problems.				
18	The history of mathematics reveals the developmental stages of its concepts.				
19	Mathematical knowledge is fixed and cannot be changed.				
20	Mathematics helps us understand better the world we live in.				
Beliefs about Mathematics Learning		**6**	**5**	**4**	**Remarks**
1	In mathematics, I can be creative and discover things by myself.				
2	In learning mathematics, I should actively participate and be involved.				
3	In order to solve math problems, I need to redo what the teacher taught.				
4	There are many possible ways or solutions to solve a mathematics problem.				
5	Understanding mathematics can help me in many ways in my day-to-day life.				
6	There are just people who are born good in math.				
7	In mathematics, I can follow its rules but I can also invent my own way of solving problems.				

8	Learning mathematics can make me understand the world I live in.				
9	I can be good in mathematics with more time, effort and practice exerted.				
10	In doing mathematics, more concepts and ideas can be created.				
11	To learn mathematics, I just need to memorize the formulas.				
12	To better understand mathematics, I should relate it to my daily activities and experiences.				
13	In understanding mathematics, other ways of solutions or proofs should be explored.				
14	In learning mathematics, I follow the rules and apply the formulas without the need to comprehend and analyze them.				
15	The more I learn and understand mathematics, the more I appreciate the world we live in.				
16	Mathematics is more of repeating what has been demonstrated by the teacher.				
17	In learning mathematics, it helps to be curious and analytical.				
18	For as long as the answer is correct, figuring out how it was solved and why the method works are unnecessary.				
19	To understand mathematics more, it is important to learn and explore the history of mathematics concepts.				
20	Learning mathematics can help me in my day-to-day decision making and reasoning.				

IMPORTANCE	RELEVANCE	CLARITY
3 – Essential	4 – Very relevant	4 – Very clear
2 – Useful but not essential	3 – Relevant but need minor revision	3 – Clear but need minor revision
1 – Not Necessary	2 – Item need some revision	2 – Item need some revision
	1 – Not relevant	1 – Not clear

APPENDIX G

STUDENT MATHEMATICS CONCEPTION INVENTORY (SMCQ)

Final Form

This questionnaire is intended to determine your level of understanding of the nature of mathematics and beliefs in mathematics learning. Please take sufficient time in reflecting your answer for each item. Kindly rate each item honestly and mindfully using the following scale:

6 – Strongly Agree
5 – Moderately Agree
4 – Slightly Agree
3 – Slightly Disagree
2 – Moderately Disagree
1 – Strongly Disagree

Nature of Mathematics Understanding		6	5	4	3	2	1
1	Mathematical knowledge is capable of changing, growing and expanding as new concepts are discovered or created.						
2	There are several other ways or methods in solving mathematics problems.						
3	The history of mathematics reveals the developmental stages of its concepts.						
4	Mathematical knowledge is fixed and cannot be changed.						
5	Mathematics has practical and functional use and purpose.						
6	Mathematics problems can be done correctly in only one way.						
7	Mathematics is a product of human exploration and curiosity.						
8	There are many possible ways or solutions to solve a mathematics problem.						
9	In mathematics, it helps to be curious and analytical.						
10	Mathematics is a tool that other disciplines such as science use to advance their concepts.						
11	Existing mathematical knowledge can be changed, revised or expanded.						
12	Mathematics is a science of pattern finding and generalizations.						
13	In understanding mathematics, other ways of solutions or proofs should be explored.						
Beliefs about Mathematics Learning and Use		**6**	**5**	**4**	**3**	**2**	**1**
1	Understanding mathematics can help me through many ways in my day-to-day life.						
2	Mathematics concepts exist to help man improve his life.						
3	Learning mathematics can make me understand the world I live in.						
4	In mathematics, I can be creative and discover things by myself.						
5	To better understand mathematics, I should relate it to my daily activities and experiences.						
6	Mathematics helps us understand better the world we live in.						
7	To understand mathematics more, it is important to learn and explore the history of mathematics concepts.						
8	Abstract and complex mathematical concepts are later on used for practical applications.						
9	New math concepts may be discovered or created that contrast previously held truths.						
10	The more I learn and understand mathematics, the more I appreciate the world we live in.						
11	In doing mathematics, more concepts and ideas can be created.						
12	In mathematics, I can follow its rules but I can also invent my own way of solving problems.						
13	Learning mathematics can help me in my day-to-day decision making and reasoning.						

MATHEMATICS LEARNING EXPERIENCE INVENTORY (MLEI)
Final Form

This questionnaire aims to determine your learning experiences appreciation and mathematics learning strategies to be able to draft better mathematics teaching and learning activities. As you go over each item think about your FCM Mathematics class. Please read each item carefully and reflect on the best scale to rate your agreement or disagreement to each statement. Please take your time in answering this questionnaire. Kindly use the following scale in rating:

6 – Strongly Agree
5 – Moderately Agree
4 – Slightly Agree
3 – Slightly Disagree
2 – Moderately Disagree
1 – Strongly Disagree

Learning Appreciation		6	5	4	3	2	1
1	I find most of what I learned in our math course very interesting and useful.						
2	I enjoy getting involved in math activities where I can work with my classmates.						
3	I appreciate the way our math lessons and activities are designed.						
4	I tend to get bored and lose interest in the middle of our math class.						
5	Our math class gives me more confidence in learning and doing math.						
6	I like that our teacher is very enthusiastic in discussing our math lessons and activities.						
7	I don't think I learned so much from our math course.						
8	I like that our teacher gives us the opportunity to solve math problems in our own way.						
9	I like that there is a motivating part in our math class where we are encourage to think and learn on our own.						
10	I like that there are opportunities in our math class where we work with co-learners.						
11	The performance feedback of my teacher and classmates help me improve my learning and understanding of mathematics.						
12	I like that our teacher discusses specific math procedures but allows us to solve problems using different methods.						
13	I learned recent and interesting things about math which I did not know before.						
14	I like that our math class has many unique and interesting activities.						
15	Through our math lessons and activities, I gained more confidence in dealing with math.						
Mathematics Learning Strategies		**6**	**5**	**4**	**3**	**2**	**1**
1	Every time I solve math problems, I go over my answer again to see if it make sense.						
2	I try to redo sample math problems and see if I can solve them using other methods.						
3	When I study math lessons, I simply read and memorize the concepts and procedures.						
4	Every time I encounter a new math concept, I try to research more about.						

5	If I don’t understand a math lesson well, I will try a different approach like research and watch tutorial videos about it.						
6	Every time I don’t solve math problems, I often have trouble making sense of the concept.						
7	In studying for mathematics, I provide sufficient time.						
8	I try to look for simpler ways of solving a math problem.						
9	To understand better math concepts, I try to relate it to real life situations and apply it into practical use.						
10	When I can’t solve a math problem after a few minutes, I tend to give up on it.						
11	I solve mathematics problems over and over again to learn it.						
12	It helps me understand math more if I can see and visualize the concepts.						
13	I try to draw or make representations so I can easily understand math problems.						
14	I solve math problems which I think I can and skip those that I find very difficult.						
15	Reading more about a math concept helps me understand it better.						

STUDENT MATHEMATICS ATTITUDINAL QUESTIONNAIRE (SMAQ)
Final Form

This is not a test. There is no right or wrong answer. We would just like to know how you feel about the following statements. This questionnaire is intended to determine your attitude towards mathematics. Kindly rate each item using the scale below. Please try to be as sincere and truthful as you can. Thank You!

Rating Scale:

6 – Strongly Agree
5 – Moderately Agree
4 – Slightly Agree
3 – Slightly Disagree
2 – Moderately Disagree
1 – Strongly Disagree

Affect Dimension		6	5	4	3	2	1
1	I feel confident whenever I do math.						
2	Mathematics is the one subject I fear about.						
3	Most of the time, I fear and get overwhelmed with mathematics.						
4	I tend to skip classroom tasks and avoid getting involved in mathematics activities because I don't like math.						
5	I try to get involved in math activities whenever possible because I enjoy doing math.						
6	I feel that mathematics has no use and is a waste of time.						
7	Because I fear math, I look for other ways in completing my math tasks such as asking someone else to do it for me.						
8	I think that mathematics is very useful in my life.						
9	After learning mathematics, I tend to forget about it.						
10	I always finish my mathematics school works on time because I like accomplishing math tasks.						
Behavior Dimension		**6**	**5**	**4**	**3**	**2**	**1**
1	I am excited for math discussions and activities.						
2	To prepare for a math test, I spend long hours reviewing.						
3	I keep solving math problems until I get the correct answer and understand the method well.						
4	I allot sufficient time studying math lessons specially the difficult ones.						
5	I spend time and effort in learning difficult mathematics concepts.						
6	I try to understand and do the different solutions given by my classmates or teacher.						
7	Overall, I find mathematics and its techniques very exciting and intriguing.						
8	I check my answers to math problems and see if it makes sense.						
9	I like and enjoy most of the mathematics subjects I have taken in school.						
10	Mathematics stirs up my interest and curiosity.						
Cognition Dimension		**6**	**5**	**4**	**3**	**2**	**1**
1	Mathematics improves one's way of thinking and reasoning.						
2	Mathematics is used from the simplest arithmetic calculations in life to more profound applications.						
3	It is important to understand and appreciate mathematics.						
4	Mathematics can be learned by everyone through persevering.						
5	Mathematics makes our lives better and easier.						
6	Mathematics can help me in my everyday life.						
7	I know mathematics is very useful in ones' life.						
8	Mathematics is used in other fields like science, engineering and medicine.						
9	I can learn mathematics with perseverance and hard work.						
10	I can apply the concepts and knowledge I learn from my mathematics classes in my day-to-day life.						

APPENDIX H

BULACAN STATE UNIVERSITY
Malolos City, Bulacan
COLLEGE OF SCIENCE

TABLE OF SPECIFICATIONS
Midterm Examination

COURSE CODE : MATH 213B
COURSE DESCRIPTION: Fundamental Concepts of Mathematical Structure

Topics	No. of Hours	Skills					Total	Placement of Items	Percent of Items
		Remembering	Understanding	Applying	Analyzing	Synthesizing			
Propositions	3 hours	1	3	2	3	1	10	1,2,4,6,7,12,13,16,17,20	20%
Tautologies and Logical Equivalence	3 hours		1	2	3	3	9	8,9,10,11,14,18,22,23,46	18%
Arguments and Proofs	4.5 hours	1	1		2	3	6	25,26,27,28,29,30	12%
Sets and Operations	3 hours	6	2	4	2	3	13	19,24,31,32,34,36,37,38, 40,44,45,47,48	26%
Set Identities and Laws of Algebra of Sets	1.5 hours	1	1	1		3	5	21,33,35,41,42	10%
Divisibility and Euclidean Algorithm	3 hours	3		3		3	7	3,5,15,39,43,49,50	14%
						TOTAL	50		100%

APPENDIX I

MATHEMATICS ACHIEVEMENT TEST
FINAL FORM

BULACAN STATE UNIVERSITY
Malolos City, Bulacan
COLLEGE OF SCIENCE

PRE TEST
Fundamental Concepts of Mathematical Structures

Name ______________________________________ Score
Surname, First Name M.I
Yr. & Sec. ____________________________________ Date ___________

Direction: Write the letter of your answer in the space provided before each number. Use black ink pen only. If your answer is not among the options write it on the blank. **Show your complete and clear solution on the space provided for each number.** Strictly no erasure. Use CAPITAL LETTER. Do your best and be honest.

______ 1. Which of the following statements is not a proposition?
A. Is 179 a composite or prime number?
B. A polygon is a closed figure having n number of side.
C. The only positive integer that divide 7 are 1 and 7 itself.
D. The integer 0 is a positive number.

______ 2. What is the truth value of the statement below which is a proposition?
Do I go to school everyday?
There are 7 days in a week.
See me on Tuesday.
In a weekly basis.
A. True
B. False
C. Both
D. Cannot be determined

______ 3. Which of the following is not true?
A. $-3|9$
B. $4 \nmid 5$
C. $21|30$
D. 195 is divisible by 13

______ 4. Which of the following statements is false?
A. $5 < 9$ and $9 < 7$
B. It is not the case that $(5 < 9\ and\ 9 < 7)$
C. $5 < 9$ or it is not the case that $(9 < 7\ and\ 5 < 7)$
D. If $9 > 7$ and $7 > 5$, then $9 > 5$

______ 5. What is the GCD(154, 803)?
A. 11
B. 22
C. 33
D. 44

_______ 6. What is the logical translation of the statement, "None of my friends *(f)* are perfect *(p)."*?

A. $f \rightarrow p$
B. $\sim f \rightarrow p$
C. $f \wedge \sim p$
D. $\sim f \vee p$

_______ 7. What is the correct translation of the statement*, "Some real numbers are rational."* into mathematical logic? Let p = Some real numbers and q = rational

A. $p \vee q$
B. $p \rightarrow q$
C. $p \wedge q$
D. $\sim p \rightarrow q$

_______ 8. The propositional logic $p \leftrightarrow q$ is equivalent to

A. $\sim(p \vee q) \wedge \sim (q \vee p)$
B. $(\sim p \vee q) \wedge (\sim q \vee p)$
C. $(p \vee q) \wedge (q \vee p)$
D. $\sim (p \vee q) \rightarrow \sim(q \vee p)$

_______ 9. Given that

$I. (p \rightarrow q) \wedge (q \rightarrow p)$
$II. p \rightarrow q$
$III. p \leftrightarrow q$
$IV. (p \rightarrow q) \vee (q \rightarrow p)$

Which of the logical statements above are equivalent?

A. I and IV
B. II and III
C. III and IV
D. I and III

_______ 10. Formulate the symbolic expression $\sim p \wedge (q \vee r)$ into words using the following:

p : Today is Monday
q : It is raining.
r : It is hot.

A. It is not Monday today or it is raining and hot.
B. Today is not Monday and it is raining and hot.
C. It is not Monday today and it is not going to rain so it is hot.
D. Today is not Monday and the weather is either rainy or hot.

_______ 11. What is the contrapositive of the conditional statement, *"If the area of a square is 169 sq.m, then its side is 13m."*?

A. If the side of a square is 13m, then its area is 169 sq.m.
B. If the side of a square is not 13m, then its area is not 169 sq.m.
C. If the area of a square is not 169 sq.m, then its side is not 13m.
D. If a polygon has equal sides of 13m, then it is a square.

_______ 12. What is the LCM of 952 and 1445 given that GCD(952,1445)=17?

A. 56
B. 85
C. 80920
D. 1375640

_______13. Represent the statement, *"If it is not the case that (6 < 6 and 7 is not less than 10), then 6 < 6"*, when $q: 7 < 10$ and $r: 6 < 6$.

A. $[\sim(r \wedge \sim q)] \rightarrow r$
B. $(\sim r \wedge \sim q) \rightarrow r$
C. $(r \wedge \sim q) \rightarrow r$
D. $[\sim(r \wedge q) \rightarrow r]$

_______14. Does the truth table of the proposition $(p \vee q) \wedge \sim (p \wedge q)$ models an exclusive or?

A. No
B. Yes
C. Sometimes
D. Cannot be determined

_______15. The propositional statement $(p \rightarrow q) \vee (p \rightarrow \sim q)$ is

A. Valid
B. Satisfiable
C. Contradiction
D. None of the above

_______16. Let $A = \{(1,0),(0,2),(-1,0),(0,-2)\}$ and $B = \{(0,0),(2,0),(0,-1)\}$, what is $A \oplus B$?

A. $A \oplus B = \{(0,0)\}$
B. $A \oplus B = \{(-1,0),(0,2)\}$
C. $A \oplus B = \{(1,0),(0,-2)\}$
D. $A \oplus B = \{(0,0),(1,0),(0,-2)\}$

_______17. Is the identity $(A \cap B) \cup (A \cap B^c) = A$ true?

A. Yes
B. No
C. Both
D. None of the above

_______18. The propositional statement $\sim p$ is logically equivalent to which statement below?

A. $\sim p \rightarrow (q \wedge \sim q)$
B. $\sim p \rightarrow (q \wedge q)$
C. $p \rightarrow (q \wedge \sim q)$
D. $p \rightarrow (q \vee \sim q)$

_______19. What is the formalize form of the sentence, *Queenie comes to the party provided that Rhianna doesn't come, but, if Rhianna comes, then Penelope doesn't come"*, given that:

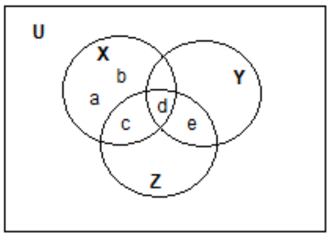
P = Penelope comes to the party
Q = Queenie comes to the party
R = Rhianna comes to the party

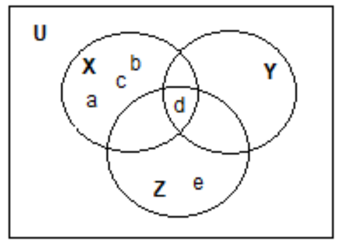
A. $(\sim R \rightarrow Q) \vee (R \rightarrow \sim P)$
B. $(R \rightarrow \sim Q) \wedge (R \rightarrow \sim P)$
C. $(R \rightarrow \sim Q) \vee (R \rightarrow \sim P)$
D. $(\sim R \rightarrow Q) \wedge (R \rightarrow \sim P)$

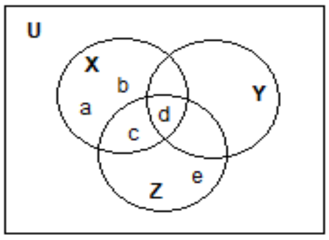
_______20. What is the correct Venn diagram representation of the relation of the sets given that

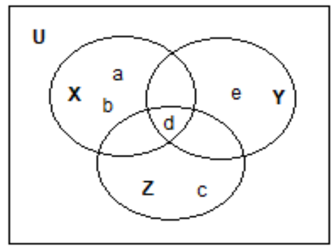
$$X = \{a, b, c, d\}$$
$$Y = \{d\}$$

$$Z = \{c, d, e\}$$

A.


B.


C.


D.


_______ 21. Using methods of deduction, will the following formal proposition $P \rightarrow Q, Q \rightarrow R, R \rightarrow S, \sim S$ and $P \vee T$ derive the conclusion T?

A. Yes
B. No
C. Both
D. Cannot be determined

_______ 22. Using propositional identities determine whether the statement, $(p \wedge q) \rightarrow (p \vee q) \equiv 1$ is true?

A. Yes
B. No
C. Both
D. Cannot be determined

_______ 23. Aladdin finds two trunks A and B in a cave. He knows that each of them either contains a treasure or a fatal trap. 33

On trunk A is written: “At least one of these two trunks contains a treasure.”
On trunk B is written: “In A there is a fatal trap.”
Aladdin knows that either both the inscription are true, or they are both false.

Can Aladdin choose a trunk being sure that he will find a treasure? If this is the case, which trunk should he open?

A. Yes. Aladdin should choose trunk A being sure it contains a treasure.
B. Yes. Aladdin should choose trunk B being sure it contains a treasure.
C. No. Aladdin should not open any of the two trunks.
D. None of the above.

_______ 24. Which of the following diagram below is the correct argument diagram of the statement, *“(1) Some students absent today are unprepared for this test, since (2) the law of averages dictates that only 10% of students absent due to illness, and (3) more than 10% are absent.”*?

A. $\dfrac{(1)+(2)}{(3)}$

B. $\dfrac{(2)+(3)}{(1)}$

C. $\dfrac{(1)\quad(3)}{(2)}$

D. $\frac{(2)\ \ (3)}{(1)}$

_______ 25. Which of the following sets is the complete list of the set $D = \{x/x \in \mathbb{N}, prime\ and\ 1 < x < 25\}$?

A. $D = \{2, 3, 4, 5, 7, 11, 12, 13, 17, 19\}$
B. $D = \{2, 3, 5, 6, 7, 11, 13, 17, 19, 23\}$
C. $D = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$
D. $D = \emptyset$

_______ 26. Given that sets A and B are well-defined sets. Which of the following statements below are correct?

I. $\emptyset \subseteq A$
II. $A\ x\ B = B\ x\ A$
III. $A \cup B\ \neq B \cup A$
IV. $A^c \cup B^c = (A \cap B)^c$

A. I and II
B. II and III
C. I and IV
D. III and IV

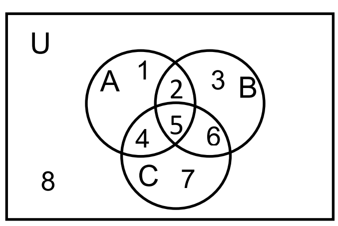
_______ 27. Given that set $R = \{0, 2, 4, 6, 8, 10\}$. How many subsets does R has?

A. 8
B. 16
C. 32
D. 64

_______ 28. In a class of 40 students, 12 enrolled for both English and Nihongo, and 22 enrolled Nihongo. If the students of the class enrolled for at least one of the two subjects, how many students enrolled for English only?

A. 4
B. 6
C. 10
D. 18

For numbers 36-38. Given the Venn Diagram of well-defined sets and their relationships below,



_______ 29. What is $B \cup C$?

A. $B \cup C = \{2, 3, 4, 6, 7\}$
B. $B \cup C = \{3, 7\}$
C. $B \cup C = \{2, 3, 4, 5, 6, 7\}$
D. $B \cup C = \{2, 5\}$

_______ 30. What is the set $(A \cup B)^c$?

A. $(A \cup B)^c = \{7, 8\}$
B. $(A \cup B)^c = \{1, 2, 3, 4, 5, 6\}$
C. $(A \cup B)^c = \{1, 3, 4, 6\}$
D. $(A \cup B)^c = \{2, 7, 8\}$

_______31. What is the set $B - C$?

A. $B - C = \{2, 3, 5\}$
B. $B - C = \{3, 5\}$
C. $B - C = \{2, 3, 5, 6\}$
D. $B - C = \{2, 3\}$

_______ 32. Is the statement $11|31674$ true?

A. Yes
B. No
C. Both
D. None of the above

_______33. Given that of the 200 freshmen BS Math students who were interviewed, 100 have a laptop of their own, 70 have desktop personal computer and 140 have a tablet gadget. There are 40 students who have both laptop and PC, 30 have both PC and tablet, 60 have both laptop and tablet, and 10 have all those three. How many students have none of three?

A. 10
B. 18
C. 20
D. 25

_______ 34. Which of the following is/are correct given that A is a well-defined set?

I. $(A^c)^c = U - A$
II. $|P|$ = cardinality of the power set of P defined as n^2
III. $A^c = \{x \in U \ / \ x \notin A\}$
IV. $A - B = A \cap B^c$

A. I and II
B. II and III
C. I and IV
D. III and IV

_______ 35. What are the values of μ and v so that $361\mu + 418v = d$, where d is the greatest common divisor of 361 and 418?

A. $\mu = 7$ and $v = -6$
B. $\mu = 6$ and $v = -7$
C. $\mu = -6$ and $v = 7$
D. $\mu = 19$ and $v = 19$

_______ 36. Given that $A = \{x/x \ is \ \ a \ positive \ even \ integer \ less \ than \ 10 \ \}, B = \{0\}$ and $C = \{2, 8\}$. Which of the following statements are correct?

I. $B \subseteq A$
II. $C \subseteq A$
III. $A \cap C = \{2, 8\}$
IV. $A \cap B = \{0\}$

A. I and II
B. II and III
C. III and IV
D. II and IV

_______ 37. Given that $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.

$A = \{a \in U/a \ is \ an \ odd \ number\}$
$B = \{b \in U/b \ is \ divisible \ by \ 5\}$

$C = \{c \in U / c \text{ is a prime number}\}$

What are the elements of $(B \oplus C) - A$?

A. $\{0, 2, 10\}$
B. $\{2, 3, 7, 10\}$
C. $\{1, 3, 5, 7, 9\}$
D. $\{2, 10\}$

_______ 38. Let $P = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14\}$ where $a_i = i$, what bit string represents the subset of all even integers P?

A. 10101010101010
B. 110110110110110
C. 00100100100100
D. 01010101010101

_______ 39. Given that 75 drink orders are made in a cafe for breakfast, and there are two types of drinks: orange juice and milk. If 59 people drink orange juice and 18 people drink milk, how many people drank both milk and orange juice?

A. 2
B. 16
C. 57
D. 73

_______ 40. If one person claps every other beat and another person claps every 3rd beat, what is the first beat they will both clap?

A. 6
B. 12
C. 24
D. 48

APPENDIX J

Table for Kuder-Richardson Computation (Final Draft)

Items	Correct	P*i*	qi	product	Scores	(x-x)2
1	39	0.65	0.35	0.2275	14	68.56
2	28	0.47	0.53	0.2489	20	5.20
3	16	0.27	0.73	0.1956	16	39.44
4	46	0.77	0.23	0.1789	23	0.52
5	41	0.68	0.32	0.2164	36	188.24
6	9	0.15	0.85	0.1275	22	0.08
7	12	0.20	0.80	0.1600	18	18.32
8	7	0.12	0.88	0.1031	24	2.96
9	27	0.45	0.55	0.2475	8	203.92
10	52	0.87	0.13	0.1156	17	27.88
11	34	0.57	0.43	0.2456	28	32.72
12	27	0.45	0.55	0.2475	19	10.76
13	38	0.63	0.37	0.2322	23	0.52
14	31	0.52	0.48	0.2497	20	5.20
15	37	0.62	0.38	0.2364	23	0.52
16	27	0.45	0.55	0.2475	34	137.36
17	39	0.65	0.35	0.2275	25	7.40
18	17	0.28	0.72	0.2031	23	0.52
19	27	0.45	0.55	0.2475	22	0.08
20	60	1.00	0.00	0.0000	23	0.52
21	38	0.63	0.37	0.2322	29	45.16
22	39	0.65	0.35	0.2275	26	13.84
23	26	0.43	0.57	0.2456	22	0.08
24	20	0.33	0.67	0.2222	24	2.96
25	55	0.92	0.08	0.0764	31	76.04
26	33	0.55	0.45	0.2475	20	5.20
27	43	0.72	0.28	0.2031	19	10.76
28	32	0.53	0.47	0.2489	17	27.88
29	58	0.97	0.03	0.0322	21	1.64
30	58	0.97	0.03	0.0322	35	161.80
31	51	0.85	0.15	0.1275	19	10.76
32	47	0.78	0.22	0.1697	25	7.40
33	33	0.55	0.45	0.2475	22	0.08
34	28	0.47	0.53	0.2489	27	22.28
35	17	0.28	0.72	0.2031	25	7.40
36	28	0.47	0.53	0.2489	33	114.92
37	40	0.67	0.33	0.2222	20	5.20
38	52	0.87	0.13	0.1156	28	32.72
39	25	0.42	0.58	0.2431	21	1.64
40	41	0.68	0.32	0.2164	21	1.64
				7.7683	24	2.96
					26	13.84
					28	32.72
					25	7.40
					31	76.04
					21	1.64

$$\rho_{KR20} = \frac{60}{60-1}\left(1-\frac{7.77}{31.77}\right)$$

$$\rho = .768$$

reliable test

	12	105.68
	13	86.12
	26	13.84
	16	39.44
	25	7.40
	19	10.76
	12	105.68
	18	18.32
	24	2.96
	16	39.44
	22	0.08
	20	5.20
	14	68.56
	22	0.08
	22	0.08
SUM	**1359**	**1938.26**
Mean	**22.278689**	**31.77**

APPENDIX K

Sample Reflection Exit Card

Reflection Exit Card No. 3 *SET THEORY*

Name: ____

Course, Year & Sec.:___________________________________ Date: __________

A. Using Venn-Euler diagram, verify the following identities. a. $A = (A \cap B) \cup (A - B)$ b. If A and B are finite sets, we have $\|A \cup B\| = \|A\| + \|B\| - \|A \cap B\|$	
CHOOSE YOUR OWN ADEVENTURE!	
Direction: Choose one from the following which interests you. Show your complete and clear solution on the space provided for each item.	
Computer Gamer	Three popular computer games are Alien Invaders, Racecar Derby, and Football Deluxe. Fifty people in your neighborhood own computer games. Sixteen of them own all three games, five own only Racecar Derby, seven own only Football Deluxe, and 19 own only Alien Invasion. How many total computers games are owned in your neighborhood?
Pet Lover	There are 49 people that own pets. 15 people own only dogs, 10 people own only cats, five people own only cats and dogs, and 3 people own cats, dogs and snakes. How many total snakes are there?
Sports Enthusiast	There are 100 athletes and three different seasons when sports are offered: soccer in the fall, basketball in the winter, and baseball in the spring. Some of the athletes play only one sport, some play two sports, and some play all three. Forty people play soccer. If 15 play all three sports, five play basketball and soccer but not baseball, and 10 play soccer only, how many people play both baseball and soccer?
Movie Marathoner	There are two movies playing at a local movie theater, Amazing Fiction 3 and Math in the Stars. 68 total people went to the movie theater. If 35 people saw Math in the Stars and 10 saw bot Amazing Fiction 3 and Math in the Stars, how many people saw only Amazing Fiction 3? How many total tickets were sold at the movie theater?
Alien Curious Mind	A group of 100 aliens arrive on Starship 2000 to invade your planet. These aliens are characterized by two distinct features, their eyes and their tails. Some aliens have eyes but no tail, some have a tail but no eyes, and some have eyes and a tail. If there are 75 aliens that have eyes, and 50 aliens that have eyes and a tail, how many aliens have eyes but no tail? How many have only a tail but no eye?
C. Prove the identities below.	

Learning Journal Questions:

1. What are the new things about sets (math) that you have learned from the discussion?

2. What can you say about the paradoxes in set theory and in other branches of mathematics? Do you think math lost its absoluteness?

3. What interesting concept have you learned from today's discussion?

4. Do you enjoy working with your classmate in accomplishing the assigned task? Do you like learning with your classmate?

5. Do you think math is developmental? That it improves or changes as time pass by? And as further modernization occurs such as more advanced technology?

APPENDIX L

Mathematics Achievement Test and Quiz Items Matching

Descriptive Information of the Item Question Matching for Mathematics Achievement

	Source	Questions	Concept/Skill	Number of Hours Taught	Retention
1.	Mathematics Achievement	Which of the following statements is not a proposition? A. Is 179 a composite or prime number? B. A polygon is a closed figure having n number of side. C. The only positive integer that divide 7 are 1 and 7 itself. D. The integer 0 is a positive number.	Proposition	3	7 days
	Quiz 1	The statement "The only positive integers that divide 7 is 1 and 7" is a proposition. (True or False)			
2	Mathematics Achievement	What is the truth value of the statement below which is a proposition? Do I go to school every day? There are 7 days in a week. See me on Tuesday. In a weekly basis. A. True B. False C. Both D. Cannot be determined	Truth value	3	7
		r			

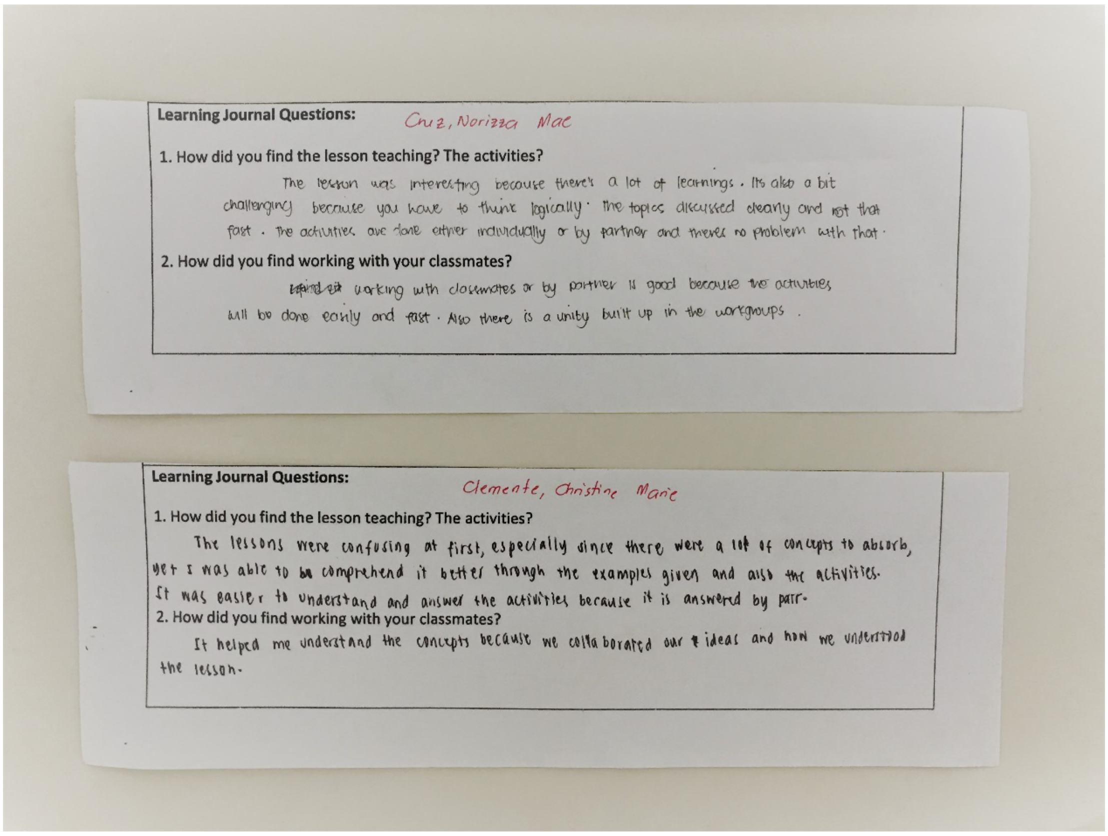
3	Mathematics Achievement	What is the logical translation of the statement, "None of my friends *(f)* are perfect *(p)*."? A. $f \rightarrow p$ B. $\sim f \rightarrow p$ C. $f \wedge \sim p$ D. $\sim f \vee p$	Translating word to symbolic logic.	3	7
	Quiz 1	Let p, q and r be the following statements: p: C++ is a computer language q: I will go to the computer shop r: I will not attend my math class Translate $\sim p \rightarrow r$ into English statement.			
4	Mathematics Achievement	Given that set $R = \{0, 2, 4, 6, 8, 10\}$. How many subsets does R has? A. 8 B.16 C. 32 D. 64	Sets and subsets	6	7
	Quiz 2	If a finite set has 9 elements, how many subsets does the set has?			
5	Mathematics Achievement	Given that of the 200 freshmen BS Math students who were interviewed, 100 have a laptop of their own, 70 have desktop personal computer and 140 have a tablet gadget. There are 40 students who have both laptop and PC, 30 have both PC and tablet, 60 have both laptop and tablet, and 10 have all those three. How many students have none of three? A. 10 B. 18 C. 20 D. 25	Sets Applications	6	7

	Quiz 2	A class of 50 took an exam in Algebra and Statistics. If 36 passed Algebra, 42 passed Statistics and 3 failed in both subjects, then how many students passed both subjects?			
6	Mathematics Achievement	What is the contrapositive of the conditional statement, *“If the area of a square is 169 sq.m, then its side is 13m.”*? A. If the side of a square is 13m, then its area is 169 sq.m. B. If the side of a square is not 13m, then its area is not 169 sq.m. C. If the area of a square is not 169 sq.m, then its side is not 13m. D. If a polygon has equal sides of 13m, then it is a square.	Converse, contrapositive, Inverse statements	3	3
	Quiz 1	Find the inverse of the conditional statement, *“If a number has a factor of 2, then it has a factor of 4.”*			
7	Mathematics Achievement	What is the LCM of 952 and 1445 given that GCD(952,1445)=17? A. 56 B. 85 C. 80920 D. 1375640	Theory of Numbers, LCM, GCD	3	7
	Quiz 3	If $\mu = 784$ and $b = 1400$, what is $\gcd(a, b)$?			
8	Mathematics Achievement	What are the values of μ and v so that $361\mu + 418v = d$, where d is the greatest common divisor of 361 and 418? A. $\mu = 7$ and $v = -6$ B. $\mu = 6$ and $v = -7$		4	3

		C. $\mu = -6$ and $v = 7$ D. $\mu = 19$ and $v = 19$	Linear Diophantine Equation		
	Quiz 3	Find a solution for $803x + 154y = 33$ for integers x and y. Give a general solution.			
9	Mathematics Achievement	Using propositional identities determine whether the statement, $(p \wedge q) \rightarrow (p \vee q) \equiv 1$ is true? A. Yes B. No C. Both D. Cannot be determined	Logical Equivalence	3	7
	Quiz 1	Determine whether $a \wedge (\sim b \vee c) \equiv a \vee (b \wedge \sim c)$ is logically equivalent			
10	Mathematics Achievement	Which of the following sets is the complete list of the set $D = \{x/x \in \mathbb{N}, prime\ and\ 1 < x < 25\}$? A. $D = \{2, 3, 4, 5, 7, 11, 12, 13, 17, 19\}$ B. $D = \{2, 3, 5, 6, 7, 11, 13, 17, 19, 23\}$ C. $D = \{2, 3, 5, 7, 11, 13, 17, 19, 23\}$ D. $D = \emptyset$	Sets	3	4
	Quiz 2	List the elements of the set $\{x: x \in \mathbb{R}, x\ is\ even\ and\ x < -1\}$.			

APPENDIX M

Sample Students' Learning Journal Entry



Learning Journal Questions: Cruz, Norizza Mae

1. How did you find the lesson teaching? The activities?

The lesson was interesting because there's a lot of learnings. Its also a bit challenging because you have to think logically. The topics discussed clearly and not that fast. The activities are done either individually or by partner and theres no problem with that.

2. How did you find working with your classmates?

~~Expand it~~ working with classmates or by partner is good because the activities will be done easily and fast. Also there is a unity built up in the workgroups.

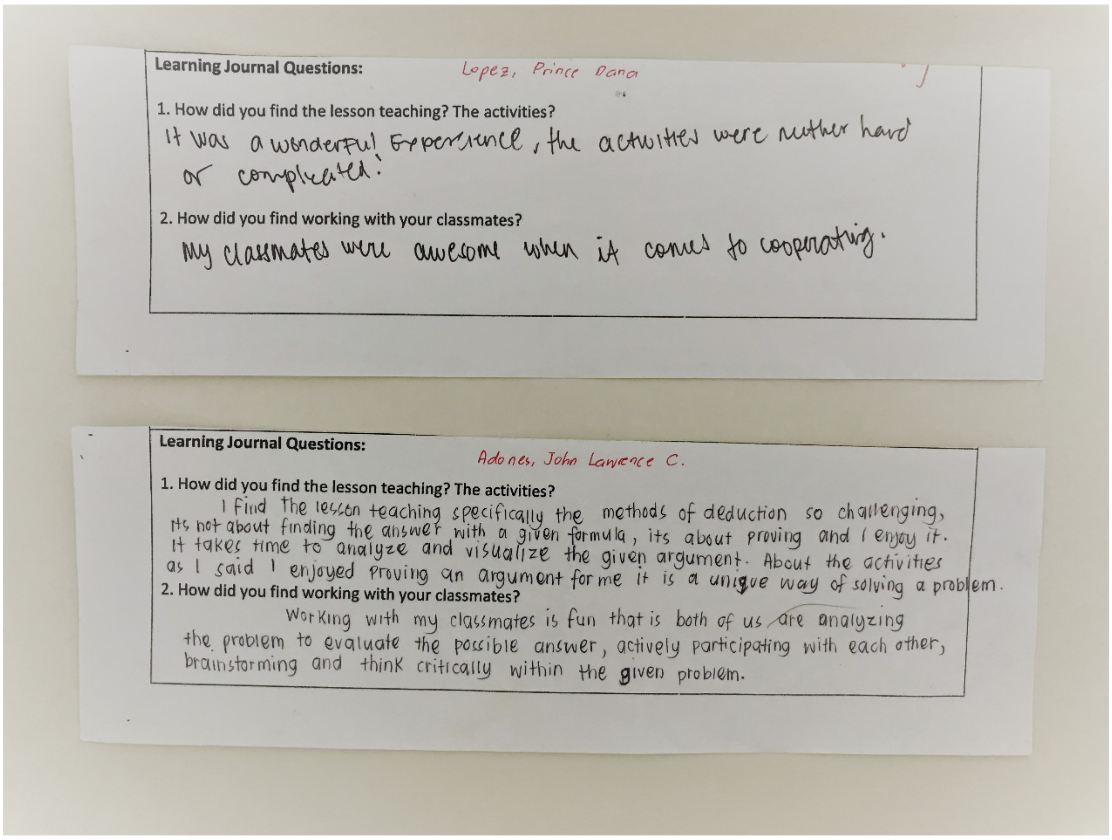
Learning Journal Questions: Clemente, Christine Marie

1. How did you find the lesson teaching? The activities?

The lessons were confusing at first, especially since there were a lot of concepts to absorb, yet I was able to ~~be~~ comprehend it better through the examples given and also the activities. It was easier to understand and answer the activities because it is answered by pair.

2. How did you find working with your classmates?

It helped me understand the concepts because we collaborated our ideas and how we understood the lesson.



Learning Journal Questions: Lopez, Prince Dana

1. How did you find the lesson teaching? The activities?

It was a wonderful Experience, the activities were neither hard or complicated!

2. How did you find working with your classmates?

My classmates were awesome when it comes to cooperating.

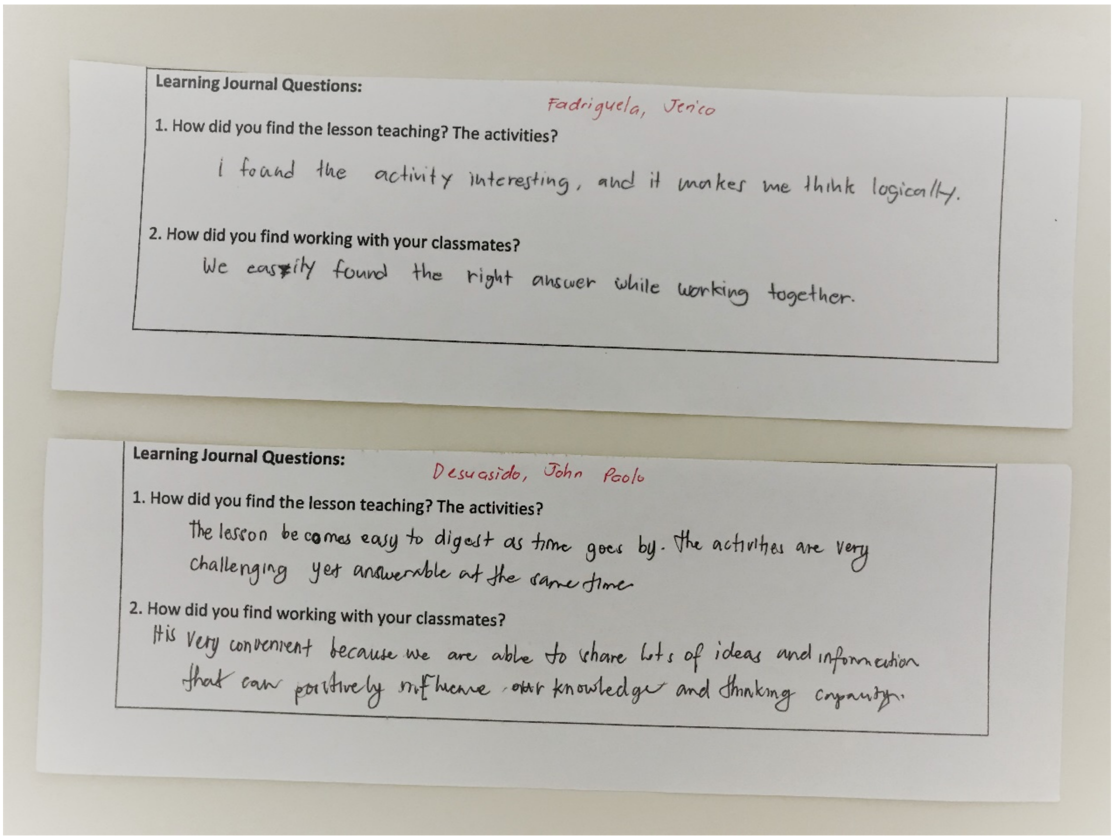
Learning Journal Questions: Adones, John Lawrence C.

1. How did you find the lesson teaching? The activities?

I Find the lesson teaching specifically the methods of deduction so challenging, its not about finding the answer with a given formula, its about proving and I enjoy it. It takes time to analyze and visualize the given argument. About the activities as I said I enjoyed proving an argument for me it is a unique way of solving a problem.

2. How did you find working with your classmates?

Working with my classmates is fun that is both of us are analyzing the problem to evaluate the possible answer, actively participating with each other, brainstorming and think critically within the given problem.



Learning Journal Questions:

Fadriguela, Jerico

1. How did you find the lesson teaching? The activities?

I found the activity interesting, and it makes me think logically.

2. How did you find working with your classmates?

We easily found the right answer while working together.

Learning Journal Questions:

Desuasido, John Paolo

1. How did you find the lesson teaching? The activities?

The lesson becomes easy to digest as time goes by. The activities are very challenging yet answerable at the same time

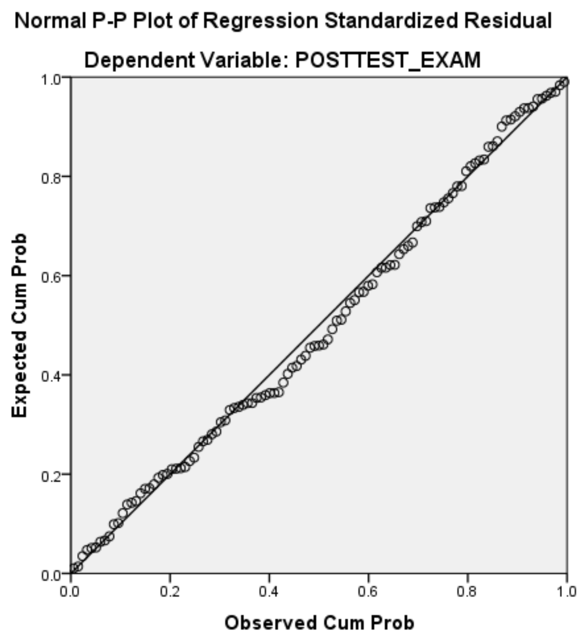
2. How did you find working with your classmates?

It's very convenient because we are able to share lots of ideas and information that can positively influence our knowledge and thinking capacity.

APPENDIX N

Multiple Linear Regression Assumptions Checking Results

Since the residuals conform to the diagonal normality line indicated in the plot as shown in the figure below, hence the residuals follow a normal distribution.



Normal P-P Plot

For homoscedasticity assumption a scatterplot of the predicted values and the residuals was graphed. Homoscedasticity refers to whether the residuals are equally distributed, or whether they compress together at some values, or spread apart in other values. Since the scatterplot in Figure 4 shows scattered points of randomly distributed data, hence the residuals is homoscedastic.

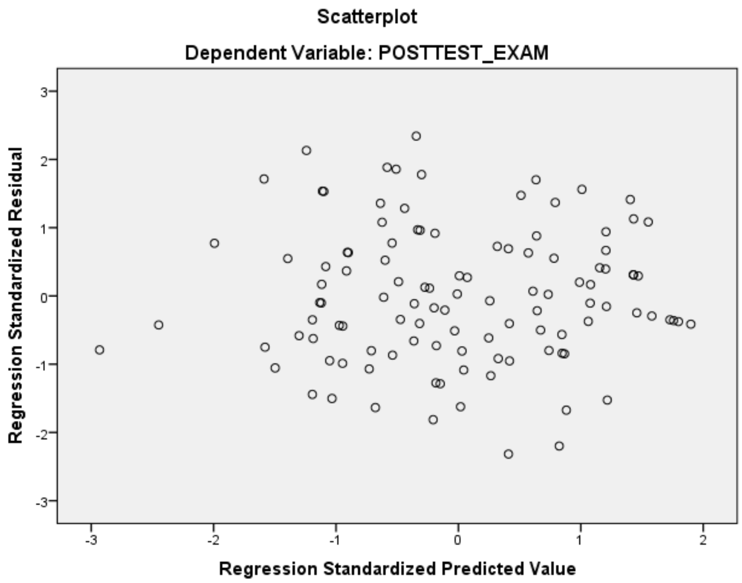


Figure 4. Scatterplot of Regression Residuals

Linearity assumption ensures that the predictor variables (independent) in the regression follows a straight-line relationship with the outcome variable (dependent). Since the residuals are normally distributed and homoscedastic, hence linearity is satisfied.

Multicollinearity assumption ensures that the predictor variables (independent) are highly correlated with each other that understanding which independent variable contributes to the variance explained in the dependent variable. This assumption was checked using Variance Inflation Factor (VIF). Table 53 presents the Regression Coefficient values.

Table 53

Regression Coefficient for Multicollinearity Assumption

Unstandardized Coefficients	Collinearity Statistics

Model	*B*	Std. Error	Standardized Coefficient Beta	*t*	Sig.	Tolerance	VIF
(Constant)	14.92	5.73		2.60	.011		
Conception	.099	.033	.316	2.99	.003	.74	1.36
Attitude	.019	.037	.054	.513	.609	.73	1.38
Learning Experiences	-.025	.025	-.090	-.987	.326	.99	1.01

Dependent Variable: Mathematics Achievement Scores

Table 53 shows that VIF values are below 5, hence the independent variables in play are not correlated to each other. Thus, the assumption for multicollinearity is met.